This follows from the arguments used in a forthcoming paper.\textsuperscript{18} It is proved by constructing an "abstract" mapping cylinder of $\lambda$ and transcribing into algebraic terms the proof of the analogous theorem on CW-complexes.

\* This note arose from consultations during the tenure of a John Simon Guggenheim Memorial Fellowship by MacLane.

\textsuperscript{18} Whitehead, J. H. C., "Combinatorial Homotopy I and II," \textit{Bull. A.M.S.}, 65, 214–245 and 453–496 (1949). We refer to these papers as CH I and CH II, respectively.

\textsuperscript{19} By a complex we shall mean a connected CW complex, as defined in §5 of CH I. We do not restrict ourselves to finite complexes. A fixed 0-cell $\phi^0 K^0$ will be the base point for all the homotopy groups in $K$.

\textsuperscript{20} MacLane, S., "Cohomology Theory in Abstract Groups III," \textit{Ann. Math.}, 50, 736–761 (1949), referred to as CT III.

\textsuperscript{21} An (unpublished) result like Theorem 1 for the homotopy type was obtained prior to these results by J. A. Zilber.

\textsuperscript{22} CT III uses in place of equation (2.4) the stronger hypothesis that $\lambda B$ contains the center of $A$, but all the relevant developments there apply under the weaker assumption (2.4).


\textsuperscript{24} Eilenberg, S., and MacLane, S., "Determination of the Second Homology . . . by Means of Homotopy Invariants," these PROCEEDINGS, 32, 277–280 (1946).


\textsuperscript{26} The hypothesis of Theorem C, requiring that $\gamma^{-1}(1)$ not be cyclic, can be readily realized by suitable choice of the free group $X$, but this hypothesis is not needed here (cf. \textsuperscript{21}).

\textsuperscript{27} Eilenberg, S., and MacLane, S., "Homology of Spaces with Operators II," \textit{Trans. A.M.S.}, 65, 49–99 (1949); referred to as HSO II.

\textsuperscript{28} $C(K)$ here is the $C(K)$ of CH II. Note that $k$ exists and is a CW complex by (N) of p. 281 of CH I and that $\phi^{-1} K^n = \tilde{K}^n$, where $\phi$ is the projection $\phi: K \rightarrow K$.

\textsuperscript{29} Whitehead, J. H. C., "Simple Homotopy Types." If $W = 1$, Theorem 5 follows from (17: 3) on p. 155 of S. Lefschetz, \textit{Algebraic Topology}, (New York, 1942) and arguments in §6 of J. H. C. Whitehead, "On Simply Connected 4-Dimensional Polyhedra" (\textit{Comm. Math. Helv.}, 22, 48–92 (1949)). However this proof cannot be generalized to the case $W \neq 1$.

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\textbf{EQUILIBRIUM POINTS IN N-PERSON GAMES}

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One may define a concept of an $n$-person game in which each player has a finite set of pure strategies and in which a definite set of payments to the $n$ players corresponds to each $n$-tuple of pure strategies, one strategy being taken for each player. For mixed strategies, which are probability
distributions over the pure strategies, the pay-off functions are the expecta-
tions of the players, thus becoming polylinear forms in the probabilities
with which the various players play their various pure strategies.

Any $n$-tuple of strategies, one for each player, may be regarded as a
point in the product space obtained by multiplying the $n$ strategy spaces
of the players. One such $n$-tuple counters another if the strategy of each
player in the countering $n$-tuple yields the highest obtainable expectation
for its player against the $n - 1$ strategies of the other players in the
countered $n$-tuple. A self-countering $n$-tuple is called an equilibrium point.

The correspondence of each $n$-tuple with its set of countering $n$-tuples
gives a one-to-many mapping of the product space into itself. From the
definition of countering we see that the set of countering points of a point
is convex. By using the continuity of the pay-off functions we see that the
graph of the mapping is closed. The closedness is equivalent to saying:
if $P_1, P_2, \ldots$ and $Q_1, Q_2, \ldots, Q_n, \ldots$ are sequences of points in the product
space where $Q_n \rightarrow Q$, $P_n \rightarrow P$ and $Q_n$ counters $P_n$ then $Q$ counters $P$.

Since the graph is closed and since the image of each point under the
mapping is convex, we infer from Kakutani's theorem\(^1\) that the mapping
has a fixed point (i.e., point contained in its image). Hence there is an
equilibrium point.

In the two-person zero-sum case the "main theorem"\(^2\) and the existence
of an equilibrium point are equivalent. In this case any two equilibrium
points lead to the same expectations for the players, but this need not occur
in general.

* The author is indebted to Dr. David Gale for suggesting the use of Kakutani's
theorem to simplify the proof and to the A. E. C. for financial support.


\(^2\) Von Neumann, J., and Morgenstern, O., The Theory of Games and Economic Be-

REMARK ON WEYL'S NOTE "INEQUALITIES BETWEEN THE
TWO KINDS OF EIGENVALUES OF A LINEAR
TRANSFORMATION"*

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Communicated by H. Weyl, November 25, 1949

In the note quoted above H. Weyl proved a Theorem involving a func-
tion $\varphi(\lambda)$ and concerning the eigenvalues $\alpha_i$ of a linear transformation $A$
and those, $\kappa_i$, of $A^*A$. If the $\kappa_i$ and $\lambda_i = |\alpha_i|^2$ are arranged in descending order,