

The Green's function is independent of this direction, however, and satisfies equations which are of the same form as (14) and (16), save for a sign change in the last term of the latter equation which arises from the different statistics associated with the integral spin field.

<sup>1</sup> Schwinger, J., *Phys. Rev.*, June 15, 1951 issue.

<sup>2</sup> We employ units in which  $\hbar = c = 1$ .

ON THE GREEN'S FUNCTIONS OF QUANTIZED FIELDS. II

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In all of the work of the preceding note there has been no explicit reference to the particular states on  $\sigma_1$  and  $\sigma_2$  that enter in the definitions of the Green's functions. This information must be contained in boundary conditions that supplement the differential equations. We shall determine these boundary conditions for the Green's functions associated with vacuum states on both  $\sigma_1$  and  $\sigma_2$ . The vacuum, as the lowest energy state of the system, can be defined only if, in the neighborhood of  $\sigma_1$  and  $\sigma_2$ , the actual external electromagnetic field is constant in some time-like direction (which need not be the same for  $\sigma_1$  and  $\sigma_2$ ). In the Dirac one-particle Green's function, for example,

$$\begin{aligned} G(x, x') &= i\langle \psi(x)\bar{\psi}(x') \rangle, x_0 > x'_0, \\ &= -i\langle \bar{\psi}(x')\psi(x) \rangle, x_0 < x'_0, \end{aligned} \tag{25}$$

the temporal variation of  $\psi(x)$  in the vicinity of  $\sigma_1$  can then be represented by

$$\psi(x) = \exp [iP_0(x_0 - X_0)]\psi(X) \exp [-iP_0(x_0 - X_0)], \tag{26}$$

where  $P_0$  is the energy operator and  $X$  is some fixed point. Therefore,

$$x \sim \sigma_1: G(x, x') = i\langle \psi(X) \exp [-i(P_0 - P_0^{\text{vac}})(x_0 - X_0)]\bar{\psi}(x') \rangle, \tag{27}$$

in which  $P_0^{\text{vac}}$  is the vacuum energy eigenvalue. Now  $P_0 - P_0^{\text{vac}}$  has no negative eigenvalues, and accordingly  $G(x, x')$ , as a function of  $x_0$  in the vicinity of  $\sigma_1$ , contains only positive frequencies, which are energy values for states of unit positive charge. The statement is true of every time-like direction, if the external field vanishes in this neighborhood.

A representation similar to (26) for the vicinity of  $\sigma_2$  yields

$$x \sim \sigma_2: G(x, x') = -i\langle \bar{\psi}(x') \exp [i(P_0 - P_0^{\text{vac}})(x_0 - X_0)]\psi(X) \rangle, \tag{28}$$

which contains only negative frequencies. In absolute value, these are the energies of unit negative charge states. We thus encounter Green's functions that obey the temporal analog of the boundary condition characteristic of a source radiating into space.<sup>1</sup> In keeping with this analogy, such Green's functions can be derived from a retarded proper time Green's function by a Fourier decomposition with respect to the mass.

The boundary condition that characterizes the Green's functions associated with vacuum states on  $\sigma_1$  and  $\sigma_2$  involves these surfaces only to the extent that they must be in the region of outgoing waves. Accordingly, the domain of these functions may conveniently be taken as the entire four-dimensional space. Thus, if the Green's function  $G_+(x, x')$ , defined by (14), (16), and the outgoing wave boundary condition, is represented by the integro-differential equation,

$$\gamma_\mu(-i\partial_\mu - eA_{+\mu}(x))G_+(x, x') + \int (dx'')M(x, x'')G_+(x'', x') = \delta(x - x'), \quad (29)$$

the integration is to be extended over all space-time. This equation can be more compactly written as

$$[\gamma(p - eA_+) + M]G_+ = 1, \quad (30)$$

by regarding the space-time coordinates as matrix indices. The mass operator  $M$  is then symbolically defined by

$$MG_+ = mG_+ + ie\gamma(\delta/\delta J)G_+. \quad (31)$$

In these formulae,  $A_+$  and  $\delta/\delta J$  are considered to be diagonal matrices,

$$(x|A_{+\mu}|x') = \delta(x - x')A_{+\mu}(x). \quad (32)$$

There is some advantage, however, in introducing "photon coordinates" explicitly (while continuing to employ matrix notation for the "particle coordinates"). Thus

$$\gamma A_+ \rightarrow \int (d\xi)\gamma(\xi)A_+(\xi), \quad (33)$$

where  $\gamma(\xi)$  is defined by

$$(x|\gamma_\mu(\xi)|x') = \gamma_\mu\delta(x - \xi)\delta(x - x'). \quad (34)$$

The differential equation for  $A_+(\xi)$  can then be written

$$-\partial_\xi^2 A_+(\xi) = J(\xi) + ie \text{Tr} [\gamma(\xi)G_+], \quad (35)$$

where  $\text{Tr}$  denotes diagonal summation with respect to spinor indices and particle coordinates. The associated photon Green's function differential equation is

$$-\partial_\xi^2 G_+(\xi, \xi') = \delta(\xi - \xi') + ie \text{Tr} [\gamma(\xi)(\delta/\delta J(\xi'))G_+]. \quad (36)$$

To express the variational derivatives that occur in (31) and (36) we introduce an auxiliary quantity defined by

$$\begin{aligned} \Gamma(\xi) &= -(\delta/\delta e A_+(\xi))G_+^{-1} \\ &= \gamma(\xi) - (\delta/\delta e A_+(\xi))M. \end{aligned} \tag{37}$$

Thus

$$(\delta/\delta J(\xi))G_+ = e \int (d\xi') G_+ \Gamma(\xi') G_+ \mathcal{G}_+(\xi', \xi), \tag{38}$$

from which we obtain

$$M = m + ie^2 \int (d\xi)(d\xi') \gamma(\xi) G_+ \Gamma(\xi') \mathcal{G}_+(\xi', \xi), \tag{39}$$

and

$$\begin{aligned} -\partial_\xi^2 \mathcal{G}_+(\xi, \xi') + \int (d\xi'') P(\xi, \xi'') \mathcal{G}_+(\xi'', \xi') &= \delta(\xi - \xi'), \\ P(\xi, \xi') &= -ie^2 \text{Tr} [\gamma(\xi) G_+ \Gamma(\xi') G_+] \end{aligned} \tag{40}$$

With the introduction of matrix notation for the photon coordinates, this Green's function equation becomes

$$(k^2 + P) \mathcal{G}_+ = 1, [\xi_\mu, k_\nu] = i\delta_{\mu\nu}, \tag{41}$$

and the polarization operator  $P$  is given by

$$P = -ie^2 \text{Tr} [\gamma G_+ \Gamma G_+]. \tag{42}$$

In this notation, the mass operator expression reads

$$M = m + ie^2 T_\rho [\gamma G_+ \Gamma \mathcal{G}_+], \tag{43}$$

where  $T_\rho$  denotes diagonal summation with respect to the photon coordinates, including the vector indices.

The two-particle Green's function

$$G_+(x_1, x_2; x_1', x_2') = (x_1, x_2 | G_{12} | x_1', x_2'), \tag{44}$$

can be represented by the integro-differential equation

$$\begin{aligned} [(\gamma\pi + M)_1(\gamma\pi + M)_2 - I_{12}]G_{12} &= 1_{12}, \\ \pi &= \not{p} - eA_+, \end{aligned} \tag{45}$$

thereby introducing the interaction operator  $I_{12}$ . The unit operator  $1_{12}$  is defined by the matrix representation

$$\begin{aligned} (x_1, x_2 | 1_{12} | x_1', x_2') &= \delta(x_1 - x_1')\delta(x_2 - x_2') - \\ &\delta(x_1 - x_2')\delta(x_2 - x_1'). \end{aligned} \tag{46}$$

On comparison with (21) we find that the interaction operator can be characterized symbolically by

$$\begin{aligned}
 I_{12}G_{12} &= -ie^2 \text{Tr}[\gamma_1\Gamma_2\mathcal{G}_+]G_{12} - ie^2 \text{Tr}[\gamma_1G_1\delta/\delta J](I_{12}G_{12}) \\
 &= -ie^2 \text{Tr}[\gamma_2\Gamma_1\mathcal{G}_+]G_{12} - ie^2 \text{Tr}[\gamma_2G_2\delta/\delta J](I_{12}G_{12}), \quad (47)
 \end{aligned}$$

where  $G_1$  and  $G_2$  are the one-particle Green's functions of the indicated particle coordinates.

The various operators that enter in the Green's function equations, the mass operator  $M$ , the polarization operator  $P$ , the interaction operator  $I_{12}$ , can be constructed by successive approximation. Thus, in the first approximation,

$$\begin{aligned}
 M(x, x') &= m\delta(x - x') + ie^2\gamma_\mu G_+(x, x')\gamma_\mu D_+(x, x'), \\
 P_{\mu\nu}(\xi, \xi') &= -ie^2 \text{tr}[\gamma_\mu G_+(\xi, \xi')\gamma_\nu G_+(\xi', \xi)], \\
 I(x_1, x_2; x_1', x_2') &= -ie^2\gamma_{1\mu}\gamma_{2\nu}D_+(x_1, x_2)(x_1, x_2|_{12}|x_1', x_2'), \quad (48)
 \end{aligned}$$

where

$$\mathcal{G}_{\mu\nu}(\xi, \xi') = \delta_{\mu\nu}D_+(\xi, \xi'), \quad (49)$$

and the Green's functions that appear in these formulae refer to the  $0$ th approximation ( $M = m, P = 0$ ). We also have, in the first approximation,

$$\begin{aligned}
 \Gamma_\mu(\xi; x, x') &= \gamma_\mu\delta(\xi - x)\delta(x - x') \\
 &- ie^2\gamma_\nu G_+(x, \xi)\gamma_\mu G_+(\xi, x')\gamma_\nu D_+(x, x') \quad (50)
 \end{aligned}$$

Perturbation theory, as applied in this manner, must not be confused with the expansion of the Green's functions in powers of the charge. The latter procedure is restricted to the treatment of scattering problems.

The solutions of the homogeneous Green's function equations constitute the wave functions that describe the various states of the system. Thus, we have the one-particle wave equation

$$(\gamma\pi + M)\psi = 0, \quad (51)$$

and the two particle wave equation

$$[(\gamma\pi + M)_1(\gamma\pi + M)_2 - I_{12}]\psi_{12} = 0, \quad (52)$$

which are applicable equally to the discussion of scattering and to the properties of bound states. In particular, the total energy and momentum eigenfunctions of two particles in isolated interaction are obtained as the solutions of (52) which are eigenfunctions for a common displacement of the two space-time coordinates. It is necessary to recognize, however, that the mass operator, for example, can be largely represented in its effect by an alteration in the mass constant and by a scale change of the Green's function. Similarly, the major effect of the polarization operator is to multiply the photon Green's function by a factor, which everywhere appears associated with the charge. It is only after these renormaliza-

tions have been performed that we deal with wave equations that involve the empirical mass and charge, and are thus of immediate physical applicability.

The details of this theory will be published elsewhere, in a series of articles entitled "The Theory of Quantized Fields."

<sup>1</sup> Green's functions of this variety have been discussed by Stueckelberg, E. C. G., *Helv. Phys. Acta*, **19**, 242 (1946), and by Feynman, R. P., *Phys. Rev.*, **76**, 749 (1949).

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### DISTRIBUTION OF MASS IN SALIVARY GLAND CHROMOSOMES

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The measurement of the absorption of soft x-rays, 8 to 12 Å in wavelength, in biological structures makes it possible to determine the total mass (dry weight) per unit area of cytologically defined areas in a biological sample. Knowing the thickness of the sample or structure being analyzed the percentage of dry substance can be estimated. For theoretical and technical details see Engström<sup>1</sup> 1950.

Dry substance is an accurate basis upon which to express the results obtained with other cytochemical techniques, e.g., the amount of specifically absorbing, ultra-violet or visible, substances.

The present investigation is an attempt to determine the dry weight (mass) of the different bands in the giant chromosomes from the cells in the larval salivary glands of the fly *Chironomus*. The structures to be observed, however, are just on the border of the resolving power of the x-ray technique for the determination of mass. The results reported, therefore, must be interpreted with care.

The specimen intended for x-ray investigation is mounted on a collodion film circa 0.5 micron thick. This film supports the object in the sample holder, a brass disk with a slit about 6 mm. long and 0.5 mm. wide. In the first experiments salivary glands from *Chironomus* were isolated on a microscope slide and the chromosomes transferred to the thin carrier membrane on the sample holder. When examining the x-ray picture of these chromosomes no details at all could be seen due to shrinkage effects when the chromosomes were dried.

For the x-ray determination of mass the specimens must be dried before they are introduced to the high vacuum of the x-ray tube. The water must also be taken away for another reason: The high absorption of soft