[11] The mathematical statement of this condition was given independently by Silver, S., *Microwave Antenna Theory and Design*, McGraw-Hill, 1949, section 39 and by Müller, Claus, *Arch. Math.*, **1** (1948–1949). Both noted that it was one of four necessary conditions for the vanishing of the integral at infinity in an existence proof based upon both tangential E and tangential H given by Stratton and Chu, *Phys. Rev.*, **56**, 99 (1939).

[12] Although terms of $F \rightarrow (1/R^3)$ as $r \rightarrow r'$ it will be found that the integral involving the regular field $\nabla \times \mathbf{E}$ vanishes by consideration of symmetry while $\nabla \times F = 0$ automatically.

ON COHOMOLOGY THEORIES*

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It is only[1] recently that the usefulness of fully normal spaces[2] in algebraic topology has been recognized. We note first that this category of spaces contains both metric and compact Hausdorff spaces.[3] Further, A. H. Stone[4] has shown that for Hausdorff spaces full normality is the same as paracompactness. Now it is well known that finiteness conditions (e.g., finite open coverings) lead to non-intuitive results for very simple spaces. In order to avoid this situation (as well as for other reasons) it is customary to introduce compactness in some form, compact supports, compact homologies and so on. This, however, introduces difficulties in applications. Very few function spaces, for example, are provided with a sufficient number of compact subsets. Many of the more interesting ones are metric and hence fully normal. It is desirable to develop a full-fledged homology theory applicable to fully normal spaces and not requiring any compactness conditions. For reasons now familiar the singular theory is inadequate. Even for locally compact connected finite-dimensional groups satisfactory results about regularity in the small have not yet been obtained in sufficient amount to permit application of the singular theory. One is then inclined toward the Čech theory (using quite arbitrary coverings) and toward cohomology rather than homology, since in the former a discrete coefficient group may be used. However, the Alexander-Kolmogoroff theory is more immediate and direct, not requiring the elaborate machinery of complexes, orientation (or ordering) and limit-groups essential to even the definition of the Čech groups. There is the additional advantage that the Eilenberg-Steenrod "axioms" (except the homotopy axiom) are known[5] to be satisfied in this theory with no restrictions at all on the spaces. Much more is known[6] when the space is fully normal. On the other hand Dowker[7] has shown that for the unrestricted Čech groups "the axioms," including the homotopy axiom, are all valid. He did not, however, prove anything beyond the weak excision theorem.

In this paper it is shown that, for fully normal spaces, the homotopy axiom holds for the Alexander-Kolmogoroff groups. Originally our proof was so devised as to apply directly to the Alexander-Kolmogoroff groups. This, however, left to one side the question of the equivalence of the Čech groups and the Alexander-Kolmogoroff groups. A positive answer was given by Spanier[8] for compact Hausdorff spaces. Our present method is this—we show that the unrestricted Čech groups are the same as the Alexander-Kolmogoroff groups. We thus slay several dragons with one stroke, obtaining Spanier's result as a corollary and getting the homotopy theorem by using Dowker's result. Looking at it from another direction we know that the extension and reduction theorems[9] hold for the unrestricted Čech groups over fully normal spaces. Hence the map excision theorem holds. Moreover we see at once that if a space has dimension at most n, then its groups in dimensions above n all vanish. Further, the groups of convex subsets of linear metric spaces all are trivial. It is thus quite plausible that one may construct an index-theory of the Leray-Schauder type for functions that are not completely continuous, since there are no compactness hypotheses in our theorems. Also it is hopeful that much of the Morse critical point theory may be developed in terms of the Alexander-Kolmogoroff groups.

Our main result is the theorem: *For fully normal spaces the Alexander-Kolmogoroff cohomology theory agrees with the unrestricted Čech cohomology theory for arbitrary coefficient groups.* A brief sketch of its proof is given as follows.

The notations of the main literature[10] will be used. In order to distinguish the Alexander-Kolmogoroff cohomology groups and the unrestricted Čech cohomology groups, we denote the latter by $\check{H}^p(X, A)$. β and γ with supplementary indices denote natural homomorphisms from cocycles to cohomology classes. Let $\sigma = (\sigma_1, \sigma_2)$ be a covering of a pair (X, A) and (K_σ, L_σ) its nerve. In the definition of $H^p(K_\sigma, L_\sigma)$ we may use[11] ordered simplexes instead of oriented simplexes. The natural homomorphism of $H^p(K_\sigma, L_\sigma)$ into $\check{H}^p(X, A)$ will be denoted by π_σ. A covering σ of (X, A) is said to be canonical if (i) $U \in \sigma_2$ implies $U \cap A \neq \phi$, (ii) $U \in \sigma_1 - \sigma_2$ implies $U - A \neq \phi$ and (iii) there is a $1 - 1$ function j_σ from the vertices of K_σ to X such that $j_\sigma(U) \in U \cap A$ or $j_\sigma(U) \in U - A$ according to $U \in \sigma_2$ or $U \in \sigma_1 - \sigma_2$. A *-refinement of a covering σ of (X, A) is a covering ρ of (X, A) such that $\rho > \sigma$, $\rho_1^* > \sigma_1$ and $(\rho_2 \wedge A)^* > \sigma_2 \wedge A$.[12] Clearly every covering of a fully normal pair (X, A) (i.e., both X and A are fully normal) has a *-refinement.

Let (X, A) be fully normal. Define a multi-valued function $\kappa: \Phi_Z^p(X, A) \to \check{H}^p(X, A)$ such that $\kappa\varphi$ consists of all the elements of the form $\pi_\sigma\gamma_\sigma j_\sigma^{\#}\varphi$, where σ is a canonical covering of (X, A) such that $\varphi = 0$ on $N_{p+1}((\sigma_2 \wedge A)^{**})$ and $\delta\varphi = 0$ on $N_{p+2}(\sigma_1^{**})$.

LEMMA 1. *κ is a homomorphism with $\kappa\Phi_B{}^p(X, A) = 0$. Hence κ induces a homomorphism $\kappa^*: H^p(X, A) \rightarrow \check{H}^p(X, A)$ such that $\kappa = \kappa^*\gamma$.*

Clearly $\kappa\varphi$ is not empty. In order to prove the single-valuedness of κ let $\pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi, \pi_\rho\gamma_\rho j_\rho{}^{\#}\varphi \in \kappa\varphi$ with $\rho > \sigma$. Let $\pi_{\rho\sigma}$: $(K_\rho, L_\rho) \rightarrow (K_\sigma, L_\sigma)$ be a projection and define g: $X \rightarrow X$ such that $g(x) = x$ for $x \neq j_\rho(V)$ and $g(j_\rho(V)) = j_\sigma\pi_{\rho\sigma}(V)$. It follows[13] that $j_\rho{}^{\#}\varphi - j_\rho{}^{\#}g^{\#}\varphi$ is a coboundary. Hence $\gamma_\rho j_\rho{}^{\#}\varphi = \pi_{\rho\sigma}{}^*\gamma_\sigma j_\sigma{}^{\#}\varphi$ and $\pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi = \pi_\rho\gamma_\rho j_\rho{}^{\#}\varphi$.

Given $\varphi, \varphi' \in \Phi_Z{}^p(X, A)$ there is some canonical covering σ of (X, A) such that $\kappa\varphi = \pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi$ and $\kappa\varphi' = \pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi'$. Hence κ is a homomorphism.

If $\varphi \in \Phi_B{}^p(X, A)$, then there is some canonical covering σ of (X, A) such that $\kappa\varphi = \pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi$ and $\gamma_\sigma j_\sigma{}^{\#}\varphi = 0$. Hence $\kappa\Phi_B{}^p(X, A) = 0$.

LEMMA 2. *If f_ρ: $(X, A) \rightarrow (Y, B)$ is a mapping, then $f^*\kappa^* = \kappa^*f^*$.*

The lemma is proved by fixing $\varphi \in \Phi_Z{}^p(Y, B)$ and letting $\kappa\varphi = \pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi$ for some canonical covering σ of (Y, B). There is a canonical covering α of (X, A) such that $\alpha > \sigma$ and $\kappa f^{\#}\varphi = \pi_\alpha\gamma_\alpha j_\alpha{}^{\#}f^{\#}\varphi$. Define $f_{\alpha\sigma}$: $(K_\alpha, L_\alpha) \rightarrow (K_\sigma, L_\sigma)$ such that $f_{\alpha\sigma}(U) = V$ implies $f(U) \subset V$. Define g: $X \rightarrow Y$ such that $g(x) = f(x)$ for $x \neq j_\alpha(U)$ and $g(j_\alpha(U)) = j_\sigma f_{\alpha\sigma}(U)$. As in Lemma 1 we have $\gamma_\alpha j_\alpha{}^{\#}g^{\#}\varphi = \gamma_\alpha j_\alpha{}^{\#}f^{\#}\varphi$ and hence $\pi_\alpha\gamma_\alpha j_\alpha{}^{\#}f^{\#}\varphi = f^*\pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi$, or $\kappa^*f^*(\gamma\varphi) = f^*\kappa^*(\gamma\varphi)$.

LEMMA 3. *$\delta\kappa^* = \kappa^*\delta$.*

We have only to consider the case $X \neq A$. Let i: $A \rightarrow X$ be the injection. For a given $\varphi \in i^{\#-1}\Phi_Z{}^p(A)$ there is a collection α_2 of open sets in X such that $\kappa i^{\#}\varphi = \pi_\sigma\gamma_\sigma j_\sigma{}^{\#}i^{\#}\varphi$ with $\sigma_1 = \alpha_2 \wedge A$. Let $\alpha_1 = \alpha_2 \cup \{X\}$ and define j_α such that $j_\alpha(U) = j_\sigma(U \cap A)$, $U \in \alpha_2$, and $j_\alpha(X) \in X - A$. Then $\kappa\delta\varphi = \pi_\alpha\gamma_\alpha j_\alpha{}^{\#}\delta\varphi$. It follows from the definition of the coboundary operator and the various permutability conditions that $\pi_\alpha\gamma_\alpha j_\alpha{}^{\#}\delta\varphi = \delta\pi_\sigma\gamma_\sigma j_\sigma{}^{\#}i^{\#}\varphi$, or $\kappa^*\delta(\beta i^{\#}\varphi) = \delta\kappa^*(\beta i^{\#}\varphi)$.

LEMMA 4. *κ^*: $H^p(X) \approx \check{H}^p(X)$.*

The idea of the proof is as follows. Given $\varphi \in \Phi_Z{}^p(X)$ with $\kappa\varphi = 0$ there is some canonical covering σ of (X, A) such that $\kappa\varphi = \pi_\sigma\gamma_\sigma j_\sigma{}^{\#}\varphi$ and $\gamma_\sigma j_\sigma{}^{\#}\varphi = 0$. Let $\rho_1{}^* > \sigma_1$ and define k_σ from X to the vertices of K_σ such that $k_\sigma(x) \in U$ implies $x \in V \subset V^* \subset U$ for some $V \in \rho_1$. Hence[13] $\gamma\varphi = \gamma(j_\sigma k_\sigma)^{\#}\varphi = 0$. Given any $\pi_\sigma\gamma_\sigma\varphi_\sigma \in \check{H}^p(X)$ let $\rho_1{}^* > \sigma_1$ and defined k_σ as above. Then $\varphi = k_\sigma{}^{\#}\varphi_\sigma \in \Phi_Z{}^p(X)$ and $\kappa\varphi = \pi_\tau\gamma_\tau j_\tau{}^{\#}\varphi = \pi_\sigma\gamma_\sigma\varphi_\sigma$, where τ is a canonical covering of (X, ϕ) with $\tau_1{}^{**} > \rho_1$.

LEMMA 5. *κ^*: $H^p(X, A) \approx \check{H}^p(X, A)$.*

The lemma is an automatic consequence of the preceding three lemmas and the exactness of cohomology sequences.

Combining Lemmas 2, 3 and 5, our theorem is proved.

After getting this result I heard that Professor Dowker obtained an analogous result.

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[1] Cartan, H., "Algebraic Topology," Harvard Univ., 1949; Wallace, A. D. "Outline for Algebraic Topology I," Tulane Univ., 1949–1950; and Wallace A. D., "Map Excision Theorem," *Duke Math. J.*, 19 (1952).

[2] Cf. Tukey, J. W., "Convergence and Uniformity in General Topology," *Ann. Math. Studies*, Princeton, 1940, p. 53.

[3] Cf. Tukey, J. W., *loc. cit.*

[4] See Stone, A. H., "Paracompactness and Product Spaces," *Bull. Am. Math. Soc.*, **54**, 977–982 (1948).

[5] Cf. Spanier, E. H., "Cohomology Theory for General Spaces," *Ann. Math.* (2), **49**, 407–427 (1948).

[6] Cf. Wallace, A. D., *loc. cit.*

[7] See Dowker, C. H., "Čech Cohomology Theory and the Axioms," *Ibid.* (2) **51**, 278–292 (1950).

[8] Cf. Spanier, E. H., *loc. cit.*

[9] Cf. Wallace, A. D., *loc. cit.*

[10] Spanier, E. H., *loc. cit.* and Dowker, C. H., *loc. cit.* (2) **51**, 418.

[11] Cf. Eilenberg, S., "Singular Homology Theory," *Ibid.*, (1944), Cor. 9.3.

[12] $\rho_2 \wedge A = \{U \cap A \mid U \in \rho_2\}$.

[13] Make use of a homotopy lemma similar to Spanier, *loc. cit.*, Lemma 9.1.

ON THE EARLY CHEMICAL HISTORY OF THE EARTH AND THE ORIGIN OF LIFE

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In the course of an extended study on the origin of the planets[1] I have come to certain definite conclusions relative to the early chemical conditions on the earth and their bearing on the origin of life. Oparin[2] has presented the arguments for the origin of life under anaerobic conditions which seem to me to be very convincing, but in a recent paper Garrison, Morrison, Hamilton, Benson and Calvin,[3] while referring to Oparin, completely ignore his arguments and describe experiments for the reduction of carbon dioxide by 40 m. e. v. helium particles from the Berkeley 60-inch cyclotron. As I believe these experiments, as well as many previous ones using ultra-violet light to reduce carbon dioxide and water and giving similar results to theirs, are quite irrelevant to the problem of the origin of life, I wish to present my views.

During the past years a number of discussions on the spontaneous origin of life have appeared in addition to that by Oparin. One of the most extensive and also the most exact from the standpoint of physical chemistry