

¹⁸ We are indebted to Dr. John D. MacLennan of the Surgery Department of Columbia University for supplying us with samples of purified collagenase and proteinase. Both enzymes solubilized the collagenous material. The collagenase appeared to be more destructive of the nerve fiber itself and of its ability to conduct impulses than was the proteinase, although this may have been primarily a concentration effect.

¹⁹ J. M. Mitchison, *Symposia Soc. Exptl. Biol.*, No. 6, p. 111, 1952.

²⁰ J. D. McColl and R. J. Rossiter, *Nature*, **166**, 347, 1950.

²¹ Rossiter (personal communication) explains that, in the paper (*ibid.*) describing the analytical results, the figures were given as milligrams per 100 gm. of sheath, whereas they should have been milligrams per 100 mg. of sheath (see also McColl and Rossiter, *J. Exptl. Biol.*, **28**, 116, 1951). The latter results for total lipid in extruded axoplasm are similar to those obtained by B. A. Koechlin in this laboratory.

²² If the connective tissue is considered to be 25 μ thick, the lipid (phospholipid plus cholesterol) would occupy a layer about 0.35 μ thick (or 15–20 of the dense layers). Electron micrographs indicate that in a sheath of this thickness many small fin fibers may be included in the "sheath" tissue analyzed, so that the above figure (0.35 μ thickness) is undoubtedly high if considered as representing only the lipid in the giant fiber Schwann cells.

²³ F. O. Schmitt and R. S. Bear, *J. Cellular Comp. Physiol.*, **9**, 261, 1937.

²⁴ J. F. Hartmann, *J. Comp. Neurol.*, **99**, 201, 1953; C. Estable, M. Reissig, and E. DeRobertis, *Exptl. Cell Research*, **6**, 255, 1954.

²⁵ F. Lipmann, *Advances in Enzymol.*, **1**, 99, 1941.

ERRATA: Fluctuations in the Space Distribution of the Galaxies

In the article of the foregoing title appearing in these PROCEEDINGS, **40**, 541–549, 1954, the following corrections should be made:

- Equation (6) should read:

$$N(m') = \omega\bar{p} \int_0^A [1 + D(r)]r^2 dr,$$

- Equation (7) should read:

$$A = 10^{1/5} [m' + 5 - M_0]$$

- The caption under Figure 1, last two lines, should read:

$$\Gamma = e^{-r^2/m^2} \text{ (solid line); } A = 10^{1/5} [m' + 5 - M_0]$$

- Equation (17) should read:

$$a = 10^{(m' + 5 - M_1)/5}$$

- Equation (19) should read:

$$N(m') = \frac{\omega\bar{p}}{M_2 - M_1} \int_{M_1}^{M_2} dM \int_0^{r'} r^2 dr [1 + D(r)]$$

- Page 545, ninth line from the bottom, equation at the end of the line, should read:

$$\Delta m = \frac{m_0 - 14.7}{5.7} \pm 0.10$$

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FLUCTUATIONS IN THE SPACE DISTRIBUTION OF THE GALAXIES

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1. *Introduction.*—In a recent paper Limber¹ has developed the equations which can be employed in an analysis of counts of galaxies to a given apparent magnitude, when the density of galaxies at a point r is described in terms of a mean density $\bar{\rho}$ and a fluctuation $D(r)$. Since the analysis requires the evaluation of the fluctuation from the mean in the number of galaxies in two directions on the sky, as a function of the angular separation of the two regions, counts of galaxies covering large continuous portions of the sky are necessary. With the exception of the data obtained by Shane,² no such extensive observational material is available. Hence it seems worthwhile to develop an analogous procedure which will make use of less extensive existing data.

Counts of galaxies in successive magnitude intervals are available³ for several hundred regions of small solid angles of the sky. From these counts, the mean square fluctuation to successive magnitude limits may be calculated. The theoretical expression relating this quantity to $D(r)$ is of interest and is derived in the following section. In section 3 the observational material is analyzed and the relevant quantities calculated. By a juxtaposition of the theoretical and calculated values, the parameters which describe the fluctuating density field may be evaluated.

2. *Derivation of the Mean Square Fluctuation.*—In the following treatment, two cases will be discussed: (i) the case in which the absolute magnitude of all galaxies is equal to M_0 , a constant, and (ii) the case in which the distribution of absolute magnitudes is an assumed function.

i. Let the density of galaxies in a given direction and at a distance r be given by $\rho(r)$. Then the number of galaxies within the small solid angle ω and distance r' is given by

$$N(r') = \omega \int_0^{r'} \rho(r)r^2 dr, \quad (1)$$

assuming space uniformly transparent. It is now assumed that the density of galaxies at any point in space may be described in terms of a mean density $\bar{\rho}$ and a fluctuation which is a function of r , $D(r)$, so that

$$\rho(r) = \bar{\rho}[1 + D(r)], \quad (2)$$

$$\overline{D(r)} = 0, \quad (3)$$

$$\overline{D^2(r)} = \beta^2 = \text{constant}, \quad (4)$$

$$\overline{D(r_1)D(r_2)} = \beta^2\Gamma(|r_1 - r_2|). \quad (5)$$

β^2 is a parameter which is equal to the mean square of the fluctuation in the density in units of the square of the density. $\Gamma(|r_1 - r_2|)$ is the correlation function of the fluctuations at the two points considered and is assumed to depend only upon the separation of the two points.

If the absolute magnitude of all galaxies is equal to M_0 , then the number of galaxies to a limiting apparent magnitude m' in the small solid angle ω is

$$N(m') = \omega\bar{p} \int_0^A [1 + D(r)]r^2 dr, \quad 1/5 [m' + 5 - M_0]. \quad (6)$$

$$A = 10. \quad (7)$$

From equation (6),

$$\overline{N(m')} = \frac{\omega\bar{p}A^3}{3}, \quad (8)$$

$$\overline{N^2(m')} = \omega^2\bar{p}^2 \int_0^A \int_0^A [1 + \beta^2\Gamma(|r_1 - r_2|)]r_1^2r_2^2 dr_1 dr_2. \quad (9)$$

Define

$$\frac{\overline{\Delta N^2}}{\overline{N^2}} = \frac{\overline{N^2(m')} - \overline{N(m')}^2}{\overline{N(m')}^2}, \quad (10)$$

whence, from equations (8) and (9),

$$\frac{\overline{\Delta N^2}}{\overline{N^2}} = \frac{9\beta^2}{A^6} \int_0^A \int_0^A \Gamma(|r_1 - r_2|)r_1^2r_2^2 dr_1 dr_2. \quad (11)$$

For each of the two forms of the correlation function,

$$a) \quad \Gamma(|r_1 - r_2|) = e^{-|r_1 - r_2|/r_0}, \quad (12)$$

$$b) \quad \Gamma(|r_1 - r_2|) = e^{-|r_1 - r_2|^2/r_0^2}, \quad (13)$$

equation (11) may be integrated twice to obtain, for a),

$$\frac{\overline{\Delta N^2}}{\overline{N^2}} = 9\beta^2 \left[e^{-A/r_0} \left\{ \frac{4r_0^4}{A^4} + \frac{8r_0^5}{A^5} + \frac{8r_0^6}{A^6} \right\} + \frac{2r_0}{5A} - \frac{8r_0^6}{A^6} - \frac{r_0^2}{A^2} + \frac{4r_0^3}{3A^3} \right]. \quad (14)$$

For $A \gg r_0$,

$$\frac{\overline{\Delta N^2}}{\overline{N^2}\beta^2} \cong \frac{18}{5} \frac{r_0}{A}. \quad (14a)$$

For b),

$$\frac{\overline{\Delta N^2}}{\overline{N^2}} = 9\beta^2 \left[-\frac{r_0^6}{15A^6} - \frac{r_0^2}{2A^2} + \left(\frac{\sqrt{\pi} r_0}{5A} + \frac{\sqrt{\pi} r_0^3}{6A^3} \right) \Phi \left(\frac{A}{r_0} \right) + \left(\frac{r_0^6}{15A^6} + \frac{r_0^4}{15A^4} + \frac{r_0^2}{5A^2} \right) e^{-A^2/r_0^2} \right], \quad (15)$$

$$\Phi\left(\frac{A}{r_0}\right) = \frac{2}{\sqrt{\pi}} \int_0^{A/r_0} e^{-A^2/r_0^2} d\left(\frac{A}{r_0}\right). \tag{16}$$

For $A \gg r_0$,

$$\frac{\overline{\Delta N^2}}{\bar{N}^2 \beta^2} \cong \frac{9}{5} \frac{\sqrt{\pi} r_0}{A}. \tag{16a}$$

r_0 is the microscale of the density distribution. In Figure 1, equations (14) and (15) are plotted as a function of A/r_0 .

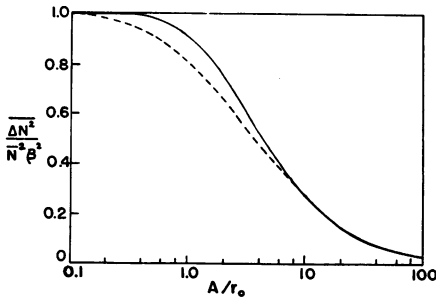


FIG. 1. The theoretical relative mean square fluctuation $\overline{\Delta N^2}/\bar{N}^2\beta^2$ in the number of galaxies counted to limiting magnitude m' for the case $\Gamma = e^{-r/r_0}$ (dashed line) and $\Gamma = e^{-r^2/r_0^2}$ (solid line); $1/5[m' + 5 - M_0]$ $A = 10$.

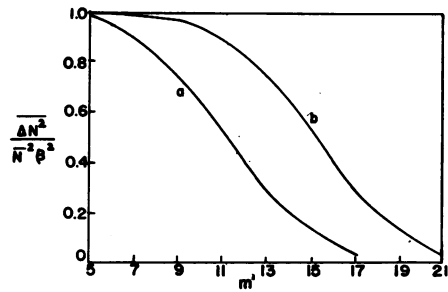


FIG. 2. The theoretical relative mean square fluctuation $\overline{\Delta N^2}/\bar{N}^2\beta^2$ in the number of galaxies counted to limiting magnitude m' for the case $\Gamma = e^{-r/r_0}$ and an assumed luminosity function for the galaxies, $N(M) = \text{constant}$, $M_1 \geq M \geq M_2$. (a) $r_0 = 2 \times 10^6$ parsecs; (b) $r_0 = 1.26 \times 10^7$ parsecs.

ii. Assume a luminosity function in which $N(m)$ is a constant, $M_1 \geq M \geq M_2$, and 0 outside this range. Let

$$a = 10^{\frac{+5 - M_1}{5}}, \tag{17}$$

$$b = 10^{\frac{m' + 5 - M_2}{5}}. \tag{18}$$

Under the above assumptions, the number of galaxies to a limiting apparent magnitude

$$N(m') = \frac{\omega \bar{\rho}}{M_1 - M_2} \int_{M_1}^{M_2} dM \int_0^{r'} r^2 dr [1 + D(r)] \tag{19}$$

may be written

$$N(m') = G \int_a^b \frac{dr'}{r^1} \int_0^{r'} r^2 dr [1 + D(r)], \tag{20}$$

$$G = \frac{\omega \bar{\rho} 5 \log_{10} e}{M_1 - M_2}. \tag{21}$$

From equation (20),

$$\bar{N} = \frac{G}{9} [b^3 - a^3], \tag{22}$$

$$\overline{\Delta N^2} = G^2 \int_a^b \int_a^b \frac{dr_1'}{r_1'} \frac{dr_2'}{r_2'} \int_0^{r_1'} \int_0^{r_2'} r_1^2 r_2^2 dr_1 dr_2 \overline{D(r_1)D(r_2)}. \tag{23}$$

Inserting the expression

$$\Gamma = e^{-|r_1 - r_2|/r_0} \quad (24)$$

and integrating first over r_1 and then over r_2 , equation (24) becomes

$$\frac{\Delta N^2}{\bar{N}^2} = \frac{9^2 \beta^2}{[b^3 - a^3]^2} \int_a^b \int_a^b \frac{dr_1'}{r_1'} \frac{dr_2'}{r_2'} \left\{ \frac{2r_0 r_2'^5}{5} + \frac{4r_0^3 r_2'^3}{3} + 2r_0^3 e^{-r_2'/r_0} \times \right. \\ \left. [r_2'^2 r_0 + 2r_2' r_0^2 + 2r_0^3] - 4r_0^6 + e^{-r_1'/r_0} e^{r_2'/r_0} [-r_0 r_1'^2 - 2r_0^2 r_1' - 2r_0^3] \times \right. \\ \left. [r_2' r_0 - 2r_2' r_0^2 + 2r_0^3] + 2r_0^3 e^{-r_1'/r_0} [r_0 r_1'^2 + 2r_0^2 r_1' + 2r_0^3] \right\}. \quad (25)$$

Because the integrand is symmetrical in r_1' and r_2' , this may be written, dropping primes,

$$\frac{\Delta N^2}{\bar{N}^2} = \frac{2 \cdot 9^2 \beta^2}{[b^3 - a^3]^2} \int_a^b \frac{dr_1}{r_1} \int_a^{r_1} \left(\frac{2r_0 r_2^4}{5} + \frac{4r_0^3 r_2^2}{3} + 2r_0^3 e^{-r_2/r_0} \times \right. \\ \left. \left[r_2 r_0 + 2r_0^2 + \frac{2r_0^3}{r_2} \right] - \frac{4r_0^6}{r_2} + e^{-r_1/r_0} e^{r_2/r_0} [-r_0 r_1^2 - 2r_0^2 r_1 - 2r_0^3] \times \right. \\ \left. \left[r_2 r_0 - 2r_0^2 + \frac{2r_0^3}{r_2} \right] + \frac{2r_0^3}{r_2} e^{-r_1/r_0} [r_0 r_1^2 + 2r_0^2 r_1 + 2r_0^3] \right) dr_2 \quad (26)$$

After two integrations, this becomes

$$\frac{\Delta N^2}{\bar{N}^2} = \frac{2 \cdot 9^2 r_0^6 \beta^2}{[b^3 - a^3]^2} \left\{ \left[4k + 2e^{a/r_0} \left(\frac{a}{r_0} - 3 \right) \right] \int_{a/r_0}^{b/r_0} \frac{e^{-x}}{x} dx + \right. \\ \left. e^{-b/r_0} \left(\frac{b}{r_0} + 3 \right) \left(2 \int_{a/r_0}^{b/r_0} \frac{e^x}{x} dx - e^{a/r_0} \left[\frac{a}{r_0} - 3 \right] - 2k \right) + \right. \\ \left. e^{-a/r_0} \left[e^{a/r_0} \left(\frac{a}{r_0} + 3 \right) \left(\frac{a}{r_0} - 3 \right) + 2k \left(\frac{a}{r_0} + 3 \right) \right] - 2k^2 + 2 \left(\frac{b}{r_0} - \frac{a}{r_0} \right) + \right. \\ \left. \frac{1}{2} \left(\frac{b^2}{r_0^2} - \frac{a^2}{r_0^2} \right) + \frac{1}{9} \left[-\frac{5}{3} \left(\frac{b^3}{r_0^3} - \frac{a^3}{r_0^3} \right) - \frac{4a^3 k}{r_0^3} \right] + \frac{2}{25} \left[\frac{1}{5} \left(\frac{b^5}{r_0^5} - \frac{a^5}{r_0^5} \right) - \right. \right. \\ \left. \left. \frac{a^5}{r_0^5} k \right] - 4 \int_{a/r_0}^{b/r_0} \frac{e^{-y}}{y} \int_{a/r_0}^y \frac{e^x}{x} dx dy \right\}. \quad (27)$$

$$k = \log_e \frac{b}{a} = (M_1 - M_2) \frac{\log_e 10}{5}. \quad (28)$$

The last term of equation (27) has been evaluated numerically. Equation (27) is plotted in Figure 2 as a function of limiting apparent magnitude m' , for two sets of values:

$$\begin{array}{lll} M_1 = -11.5, & M_2 = -18.0, & r_0 = 2 \times 10^6 \text{ parsecs,} \\ M_1 = -11.5, & M_2 = -18.0, & r_0 = 1.26 \times 10^7 \text{ parsecs.} \end{array}$$

The limits of the luminosity function have been determined so that the resulting luminosity function corresponds approximately to Holmberg's⁴ luminosity function, which has been revised for the new distance scale.

It may be noted that as $a \rightarrow b$ in equation (27), i.e., the limits of the luminosity function approach M_0 , the expression goes over to equation (14), the analogous equation derived with the assumption that $M = M_0$ for all galaxies.

3. *Observational Material.*—From publications of the Harvard College Observatory, one hundred plates are available⁵ with $|b|$ over 30° . For each plate the number of galaxies at each one-tenth magnitude interval for the central 9 square degrees of the plate has been tabulated.⁶ The distribution on the sky of these plates is shown in Figure 3.

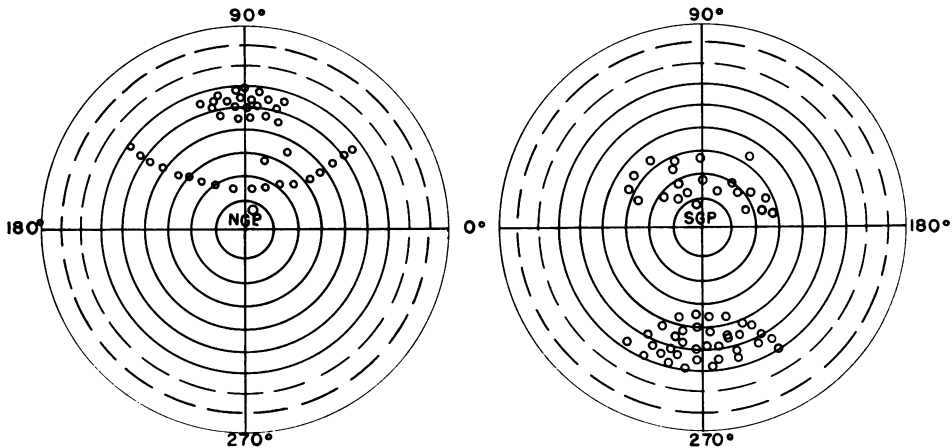


FIG. 3 Distribution of 100 Harvard plates

The following corrections have been applied to the published data.

a) The faint end of the Harvard magnitude scale appears systematically too bright by several tenths of a magnitude. This discrepancy, noted previously by Seyfert⁷ by comparing Harvard magnitudes with measures obtained by Baade,⁸ is further indicated by the fact that the logarithm of the ratio of the number of galaxies at one magnitude to that at the previous, $\log [A(m+1)/A(m)] = 0.70$, which is considerably higher than the theoretical value for uniform space distribution. While compatible with the Stebbins-Whitford effect, the amount of the excess is too great to be accounted for by an increase in the mean absolute magnitude of galaxies with increasing distance from our galaxy. Therefore, each observed magnitude m_0 has been corrected by an amount $\Delta m = \frac{(m_0 - 14.7 \pm 0.10)}{5.7}$.

The mean error has been estimated.

b) The magnitude readings of each plate have been corrected by an amount $\Delta m = \kappa \csc \beta / 0.6$ for the effects of interstellar absorption, and the resulting magnitude of each galaxy is that which would be observed outside the obscuration within our galaxy. From the number of galaxies up to magnitude 17.5 for each of the 100 plates, κ was determined as $\kappa = 0.38 \pm 0.05$.

c) No corrections for differential extinction or plate limb effects are included, for studies by Shapley and his co-workers⁹ have shown such effects to be negligible.

No red-shift corrections are applied, for the amount of such correction is negligible at the distances included in this survey.

The over-all mean error is estimated from Table 1. The counted number of galaxies N to a limiting magnitude has a mean error $\pm N/3$.

| | Amount (M.E.) | Reference |
|--|---------------|--------------------------------------|
| <i>Source of error in magnitude:</i> | | |
| Individual magnitude estimate | ± 0.017 | HA 105, No. 10; HB 895, 905; HC 423. |
| Magnitude scale correction | ± 0.10 | Estimated |
| Absorption correction | ± 0.085 | Calculated |
| Total | ± 0.21 | |
| <i>Source of error in counts of N galaxies:</i> | | |
| Magnitude error ± 0.021 | $\pm 0.33N$ | Calculated |
| Counting error | $\pm 0.05N$ | Estimated |
| Total | $\pm 0.33N$ | |

TABLE 2
VALUES OF \bar{N} , $\overline{\Delta N^2}$, AND $\overline{\Delta N^2}/\bar{N}^2$ FOR THE HARVARD PLATES

| $m'_{\text{corrected}}$ | No. Plates | \bar{N} | $\overline{\Delta N^2}$ | $\overline{\Delta N^2}/\bar{N}^2$ |
|-------------------------|------------|-----------------|-------------------------|-----------------------------------|
| 14.2 | 82 | 3.4 ± 0.18 | 13.3 ± 4.0 | 1.15 ± 0.34 |
| 14.8 | 100 | 8.2 ± 0.34 | 40 ± 12 | 0.60 ± 0.18 |
| 15.4 | 100 | 18.3 ± 0.71 | 135 ± 47 | 0.40 ± 0.14 |
| 16.6 | 99 | 92 ± 3.6 | $3,580 \pm 1,470$ | 0.42 ± 0.17 |
| 17.3 | 65 | 181 ± 8.3 | $8,790 \pm 4,650$ | 0.27 ± 0.14 |
| 17.7 | 21 | 282 ± 23 | $23,690 \pm 21,000$ | 0.30 ± 0.26 |

The homogeneity of the observational material may be examined as follows. The 100 plates are arranged according to latitude, and the mean number of galaxies at any magnitude (after correction) is determined by including counts from first

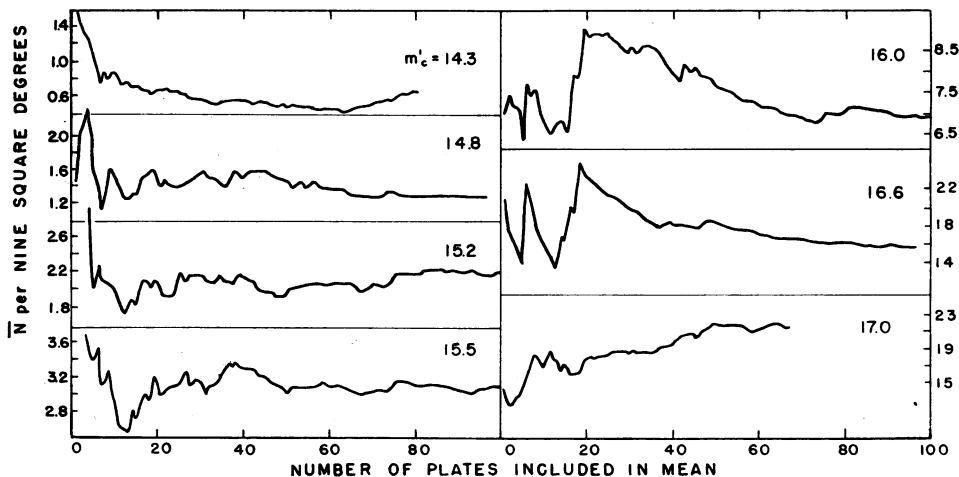


FIG. 4. The mean number of galaxies, per 9 square degrees, for selected magnitudes, as a function of the number of plates included in the mean.

one plate in the mean, then two plates, and so on, until counts for all plates are included in the mean. A plot of the mean number of galaxies against the number of

plates included in the mean indicates a leveling-off, at all magnitudes, when about 60 plates have been included. Some representative plots are shown in Figure 4. Thus it appears valid to group the data for the 100 plates without any loss of information.

Table 2 includes values of \bar{N} , $\overline{\Delta N^2}$, and $\overline{\Delta N^2}/\bar{N}^2$, and their mean errors. All values have been calculated from the counts for the central 9 square degrees of each plate.

In addition to the above material, counts of the total number of galaxies recorded to the magnitude limit of the plate are available¹⁰ for 222 plates in the two zones; $57^\circ \leq |b| \leq 75^\circ$, $76^\circ \leq |b| \leq 90^\circ$. Table 3 includes this available material. The first column is the source of the data, and the second column the arbitrary magnitude limit to which all counts have been reduced by Shapley, on the assumption of uniform space density. The third column contains the latitude ranges into which the data have been grouped for the present analysis, with the absorption correction assumed to be constant over the zone. The fourth and fifth columns list the corrected magnitude limit and the number of plates in each zone. The final three columns contain values of \bar{N} , $\overline{\Delta N^2}$, and $\overline{\Delta N^2}/\bar{N}^2$, per square degree, calculated from counts of the central 9 square degrees of each plate, to the noted magnitude limit. The final entry refers to the data of Shane, with the limiting magnitude 18.6 listed by Neyman and Scott reduced to 18.0 outside the galaxy.

TABLE 3
VALUES OF \bar{N} , $\overline{\Delta N^2}$, AND $\overline{\Delta N^2}/\bar{N}^2$ FOR THE REMAINING PLATES

| Reference | m' reduced | Latitude | m' corrected | No. Plates | \bar{N} | $\overline{\Delta N^2}$ | $\overline{\Delta N^2}/\bar{N}^2$ |
|---|--------------|----------|----------------|----------------------|------------|-------------------------|-----------------------------------|
| HR 347 | 17.5 | 58°-75° | 17.3 | 100 | 44.8 ± 1.6 | 470 ± 218 | 0.27 ± 0.11 |
| HR 347 | 17.5 | 76-90 | 17.4 | 23 | 47.5 ± 3.6 | 477 ± 499 | .21 ± .22 |
| HR 333 | 17.9 | 57-75 | 17.8 | 27 | 28.1 ± 1.9 | 134 ± 152 | .17 ± .19 |
| HC 423 | 18.2 | 58-75 | 18.1 | 53* | 53.9 ± 2.8 | 1,017 ± 585 | .35 ± .20 |
| HC 423 | 18.2 | 76-90 | 18.2 | 19 | 39.4 ± 3.2 | 246 ± 320 | .16 ± .20 |
| <i>Astrophys. J.</i> 117, 132, 1953 | .. | ... | 18.0 | 5,796 sq. degrees | 89 | 1,200 | 0.15 |

* When three high-density plates are omitted, $\bar{N} = 48.1$, $\overline{\Delta N^2}/\bar{N}^2 = 0.21$.

In Figures 5 and 6, the calculated values of $\overline{\Delta N^2}/\bar{N}^2$ are superimposed on the theoretical curve for $\overline{\Delta N^2}/\bar{N}^2\beta^2$, $\beta^2 = 1$. In Figure 5 the abscissae are $10^{m'/5}$ (*upper*) and A/r_0 (*lower*) for the calculated and theoretical values, respectively. With $\beta^2 = 1$, the following values may be read from the graphs:

$$\text{For } \Gamma = e^{-r/r_0}: \text{ at } A/r_0 = 1; 10^{m'/5} = 2.5 \times 10^2, \text{ or } m' = 12.0.$$

$$\text{For } \Gamma = e^{-r^2/r_0^2}: \text{ at } A/r_0 = 1; 10^{m'/5} = 2.5 \times 10^2, \text{ or } m' = 12.0.$$

Thus r_0 , the microscale of the fluctuations, is equal to the mean distance of a twelfth-magnitude galaxy, independent of any assumption of the mean absolute magnitude of a galaxy. If $M_0 = -14.8$, $r_0 = 2.3 \times 10^6$ parsecs.

In Figure 6, $\Gamma = e^{-r/r_0}$, $M_1 = -11.5$, $M_2 = -18.0$, $r_0 = 1.26 \times 10^7$ parsecs.

As β^2 increases, the calculated values will fall on the tail of the curve, where $A/r_0 > 1$. Due to the asymptotic form of the expressions for $\overline{\Delta N^2}/\bar{N}^2\beta^2$ [equations (14a) and (16a)], it may be seen that the variation of r_0 with β^2 is given by $r_0\beta^2 = 2.3 \times 10^6$ parsecs, or $r_0\beta^2 = 1.26 \times 10^7$ parsecs, $\beta^2 \geq 1$. It appears that the inclu-

sion of a luminosity function affects in a significant way the microscale of the fluctuations.

This analysis does not permit independent determinations of r_0 and β^2 . However, under certain conditions, estimates of r_0 may be made on physical grounds,¹⁰ and β^2 evaluated.¹¹

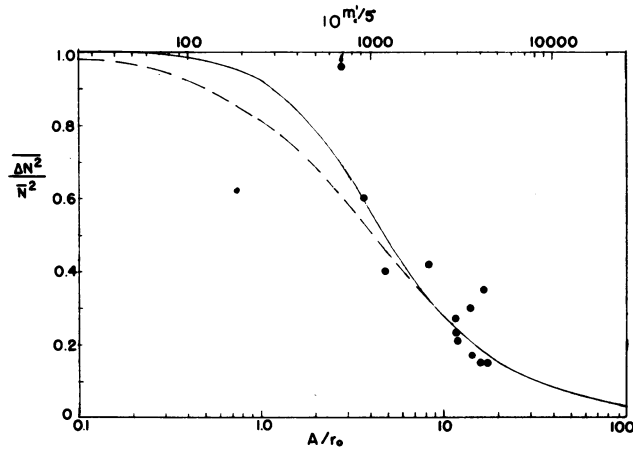


FIG. 5. The relative mean-square fluctuation $\overline{\Delta N^2}/\overline{N^2}$ in the number of galaxies counted to limiting magnitude m'_c for all available data, as a function of $10^{m'_c/5}$ (upper scale). By superimposing the observed values on the predicted curves, the quantity $r_0\beta^2$ may be evaluated. The dashed curve is derived for $\Gamma = e^{-r/r_0}$, $\beta^2 = 1$, while the solid curve is derived for $\Gamma = e^{-r^2/r_0^2}$, $\beta^2 = 1$.

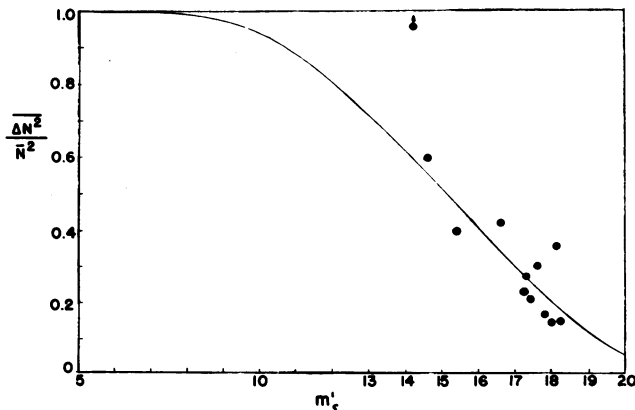


FIG. 6. The relative mean-square fluctuation $\overline{\Delta N^2}/\overline{N^2}$ in the number of galaxies counted to limiting magnitude m'_c for all available data, as a function of m'_c . The curve represents the predicted values with $r_0 = 1.26 \times 10^7$ parsecs, $\beta^2 = 1$.

4. *Conclusions.*—It is apparent that the space distribution of the galaxies may be analyzed in terms of a fluctuating density field, using methods developed from the physical theory of turbulence. The advantages of the method are twofold: it requires a minimum number of parameters, and it is physically reasonable if the

galaxies have condensed from a turbulent gaseous medium. An upper limit to r_0 , the microscale of the turbulence, is of the order of 10^7 parsecs and hence is of the order of the dimensions of clusters of galaxies.

I would like to express my thanks to Professor Gamow, for suggesting this investigation and for his continued enthusiasm throughout its course; to Dr. Frenkiel for helpful discussions on the theory of turbulence and for suggestions concerning the treatment of the data; to Fr. Heyden and Dr. Shapley for their continued interest in the problem; to Dr. Baade for a discussion of the magnitude error of the Harvard plates; and to Dr. Robert J. Rubin, with whom all phases of this work have been discussed.

¹ D. N. Limber, *Astrophys. J.*, **117**, 134–144, 1953.

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³ *Harvard Ann.*, **105**, No. 8, 1937; **105**, No. 10, 1937; **106**, No. 1, 1938; these PROCEEDINGS, **26**, 166, 1940; **26**, 554, 1940; **36**, 157, 1950; **37**, 191, 1951; *Harvard Circ.*, No. 423, 1937.

⁴ E. Holmberg, *Lund. Medd.*, ser. II, p. 128, 1950.

⁵ *Harvard Ann.*, **105**, No. 8; **105**, No. 10; **106**, No. 1; *Harvard Reprints*, Nos. 194, 208.

⁶ *Harvard Reprint* No. 208 totals frequencies for the 25 central square degrees. Shapley has, advised, in a private communication, that the tabulated values merely be reduced by the factor 9/25.

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⁸ W. Baade, *Astrophys. Nachr.*, **233**, 65, 1928.

⁹ *Harvard Ann.*, **106**, No. 1, 7, 1938.

¹⁰ G. Gamow, these PROCEEDINGS, **40**, 480–484, 1954.

¹¹ The author is indebted to Dr. Chandrasekhar for a copy of a paper by Limber to be published in the May, 1954, *Astrophysical Journal*, in which the data of Shane are analyzed in terms of a fluctuating density field. Limber's results, while more complete than those obtained here, indicate values of the same order of magnitude as the above.

MAGELLANIC CLOUDS. XIII. COMPARISON OF MAGELLANIC AND GALACTIC ECLIPSING VARIABLES*

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Mrs. Nail's observations¹ of eclipsing stars in the Magellanic Clouds provide conclusive evidence that such systems are numerous there and make it possible, for the first time, to compare them with objects of the same type in the Galaxy.

Periods, apparent photographic magnitudes, and ranges are available for forty-four Magellanic pairs. With the distance modulus 19.0² (uncorrected for space absorption), the absolute magnitudes at maximum run from -6.45 to -2.5 . The spectra of the variables have not been observed, but Shapley and Nail are convinced that they are blue stars, presumably of early B type.

Galactic eclipsing variables of comparable absolute magnitude must be selected for comparison. This must be done from the available data regarding linear diameter and surface brightness.