

but simply decay through direct viscous action.⁵ The present calculation then shows explicitly that this statistical dependence between the different Fourier coefficients does exist and persists but that, for Heisenberg's theory, it has a universal character determined only by the transfer mechanism.

¹ G. K. Batchelor, *The Theory of Homogeneous Turbulence* (Cambridge: At the University Press, 1954).

² G. K. Batchelor and A. A. Townsend, "Decay of Turbulence in the Final Period of Decay," *Proc. Roy. Soc. London, A*, 194, 527-543, 1948.

³ W. Heisenberg, "Zur statistischen Theorie der Turbulenz," *Z. Physik*, 124, 628-657, 1948.

⁴ W. H. Reid, "Two Remarks on Heisenberg's Theory of Isotropic Turbulence," *Quart. Appl. Math.* 14, 201-205, 1956.

⁵ Cf. M. J. Lighthill, *Nature*, 173, 746, 1954. I am indebted to Professor Lighthill for some further illuminating remarks regarding this point.

ON THE CLASSIFICATION OF REINFORCEMENT SCHEDULES*

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Schedules of reinforcement for operant behavior have ordinarily been classified under two major headings: those based on the passage of specified time intervals of nonreinforcement between reinforced responses and those based on the emission of specified numbers of responses by the organism. Skinner describes this distinction as one between schedules which "are arranged by a system outside the organism [fixed and variable interval schedules] and those which are controlled by the behavior itself [fixed and variable ratio schedules]."¹ Both types of schedule have in common a periodicity or intermittence of reinforcement, but some of the differences in their behavioral effects are striking.² In view of the differential effects observed on behavior and the different procedures followed by the experimenter, the two categories are regarded as distinct by most, if not all, theorists. Nevertheless, it would seem worth while if such an integration could be made, and an attempt in this direction is offered in the present paper.

We may consider that a schedule of reinforcement acts to maintain behavior through the differential reinforcement of a particular pattern of responses in time (or, equivalently, single responses having certain temporal characteristics with respect to other responses). From this viewpoint the "count" involved in a "ratio" schedule may be only incidental to the generation of a maximal rate of responding, with the crucial factor being the increased probability of reinforcement for responses following each other at short intervals. By implication, then, if external conditions were arranged to favor rapid responding by the organism, we might expect to observe the "bursts" (short periods of high response rate) and "breaks" (pauses in responding) so characteristic of "ratio" reinforcement. "Interval" schedules, on the other hand, may differentially reinforce responses preceded by relatively longer intervals of no responding, producing the response rates and temporal distributions characteristic of these schedules. Skinner has

noted that "schedules are simply rather inaccurate ways of reinforcing rates of responding."³ If these schedules are so regarded, there is no reason to believe that their integration into one conceptual framework is impossible.

The projected classification of reinforcement schedules utilizes "time-sampling" contingencies, in the description of which we shall employ the terms t^D and t^A . The term t^D represents a period of time during which a specified instrumental response of the organism may be followed by a reinforcing event; t^A represents a period of time during which this response will not be followed by a reinforcing event. A fundamental case may be established with these three restrictions: (1) t^D and t^A are held constant, (2) t^D and t^A are alternated, and (3) only the first response in t^D is reinforced. It may be noted that reinforcements can easily be "missed" on such a schedule; if the organism does not respond at all during the t^D period, it does not receive a reinforcement during that $t^D + t^A$ cycle.

The experimental domain suggested by these variables may be visualized as a co-ordinate system with the duration of t^D along one side and the duration of t^A along the other. Increasing either t^D or t^A alone changes both the total cycle length, $t^D + t^A$, and the proportion of the cycle during which a response may be reinforced, $t^D/(t^D + t^A)$. The experimental domain may then be mapped as in Figure 1, wherein are schematized certain sectors which may represent different behavioral effects correlated with classically defined "interval" and "ratio" schedules. It should be remembered that the areas and boundaries of the sectors are merely suggestive; their existence and limits (probably not sharp) remain to be empirically determined in terms of the behavioral properties exhibited.

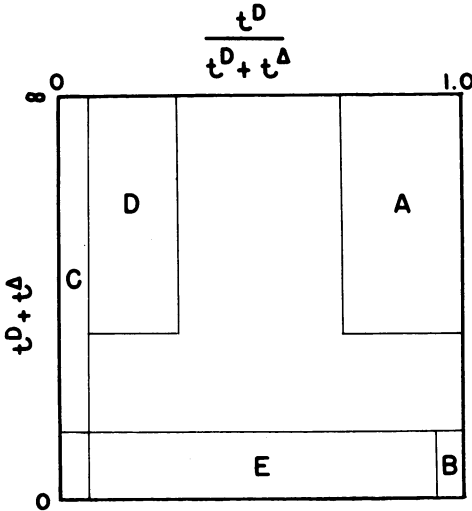


FIG. 1.—For explanation see text.

Fixed Interval Schedules.—This case is marked by $t^D/(t^D + t^A)$ approximating or equaling unity (where $t^A \rightarrow 0$ and $t^D > 0$) and comparatively long cycle lengths. If $t^D/(t^D + t^A) = 1$, the organism can "miss" a reinforcement only in the sense that a whole cycle may pass without a response, with one reinforcement opportunity lost, though the next subsequent response is reinforced whenever it occurs. When $0 < t^D/(t^D + t^A) < 1$, "missing" of a reinforcement can take place in the same way, but it can also occur when the first response in a cycle is made in t^A . With $t^D + t^A$ long enough, these schedules permit the development of a temporal discrimination, as shown by a varying "density" of response in which the organism's interresponse intervals do not remain random but systematically decrease in any cycle. These schedules are schematically located in sector A of Figure 1.

Regular Reinforcement and Extinction.—"Regular reinforcement" is obtained with $t^A = 0$ and with cycle lengths (composed entirely of t^D) shorter than the duration of a reinforcement. Thus, if a pigeon is allowed 3 seconds' access to

grain as a reinforcement, a t^D equal to 2 seconds and $t^A = 0$ will insure that a new cycle has begun by the time the bird has finished eating, and every response will be reinforced (see sector *B*, Fig. 1).

"Extinction" is the case where $t^D \rightarrow 0$ and $t^A > 0$ ($t^D/(t^D + t^A) \rightarrow 0$). The subject is reinforced so infrequently that behavior cannot be maintained (see sector *C*, Fig. 1).

Ratio Schedules.—The incorporation of response contingency ("ratio") schedules into the proposed framework apparently presents greater problems than the above, since the co-ordinates of Figure 1 are temporal in nature. Several considerations, however, help to resolve the apparent incompatibility between the two metrics of time and response number. By reducing $t^D/(t^D + t^A)$ to a sufficiently small value and keeping reasonably long cycle lengths, we favor the adoption by the organism of a high response rate, since on such a schedule reinforcement is more probable or frequent following short interresponse intervals (i.e., high rates). Sector *D* of Figure 1, by such reasoning, would include t^D and t^A values which might be expected to yield behavioral effects typical of fixed ratio and some variable ratio schedules. It is an empirical question whether choices of such t^D and t^A values would result in "ratio" behavior ("breaks" and "bursts," etc.) and whether sudden large decreases in $t^D/(t^D + t^A)$ would reproduce the known effects² of switching an organism to a much higher fixed or mean variable ratio. Some preliminary data will be presented below indicating that such is, indeed, the outcome.

Certain variable ratio schedules, which might be termed "random ratio," specify the probability of reinforcement for any particular response; a 0.25 random ratio, for example, indicates that there is a probability of 0.25 that any single response will be reinforced. The rates of responding generated by such schedules are particularly stable and are free of the "breaks" after reinforcement.⁴ In the present framework, as cycle lengths become shorter, such rapid alternations of t^D and t^A are eventually reached that "breaks" after reinforcements should disappear, because responses immediately after reinforcement are as likely to be reinforced as are any other responses. In this case, the probability of reinforcement for any response, as with "random ratio" schedules, can be specified by the proportion $t^D/(t^D + t^A)$. For example, with short cycle lengths, perhaps of the order of less than a second, and with $t^D/(t^D + t^A)$ equal to $1/20$, there is on the average one chance in twenty of a given response's being reinforced. Sector *B*, "regular reinforcement," fits into sector *E*, "random ratio reinforcement," and accounts for cases where the probability of reinforcement is unity for each response. Sector *C*, "extinction," is also included in sector *E* and covers the case where there is a probability of zero that any response be reinforced. Some preliminary results displaying "random ratio" response features, for schedules assignable to sector *E*, will be presented below.

It may be noted that "breaks" after reinforcement are observed in high "fixed ratio" schedules and "variable ratio" schedules with a high mean ratio but become negligible during random and low mean variable ratio schedules of reinforcement. We would expect a similar difference to exist between sectors *D* and *E* of Figure 1; in sector *D* (presumably fixed and high mean variable ratio schedules) the cycle length is long enough so that responses immediately after reinforcements are almost never reinforced, while in sector *E*, as remarked earlier, the short cycle

length makes "breaks" after reinforcement unlikely. Additionally, we might expect that once the cycle length goes below a critical small value, cycle length should cease to be an important variable; the organism's responding is then affected only by the $t^D/(t^D + t^A)$ value prevailing.

The areas of Figure 1 as yet unmentioned cannot now be categorized in any systematic way with regard to their behavioral effects, and they may represent schedules of reinforcement not falling within conventional categories. Empirical determinations in these areas may indicate a blending of "interval" and "ratio" behavior.

Other Schedules.—The three restrictions underlying Figure 1 are not the only possibilities. Each represents a special case of one or more variables which might themselves be systematically manipulated. For example, a schedule may be constructed in which t^D and t^A are not held constant but are varied according to some program consisting, perhaps, of a prearranged contingency table governing the probability at any time of passing from one part of the domain to another. If such a program were designed incorporating a high probability of arriving at and staying in the "interval" area of Figure 1 (sector *A*) and a low probability of staying within any single portion of that area, we should expect the resultant behavior to approximate that appropriate to "variable interval" schedules. In like manner, a program which kept the schedule of reinforcement within sector *D* of Figure 1 should yield behavior appropriate to a "variable ratio" schedule though possibly not to "random ratio." Greater complexity in behavioral properties might result from a program not limited to a particular sector of Figure 1.

We may wish, in addition, to alter the other two restrictions underlying Figure 1. For example, it would be possible to present randomly, rather than by regular alternations, a fixed t^D and a fixed t^A . Further, it would be possible to specify any desired probability of reinforcement of other than the first, or more than one, response in t^D . In fact, the entire analysis up to now could be duplicated in any t^D period by including within t^D itself any "interval" or "ratio" schedule so far discussed. Obviously, too, all the usual independent variables of behavioral research, such as drive, could figure as parameters of the present classification.

SOME EXPERIMENTAL RESULTS

With the end in mind of experimentally exploring the above schema, an apparatus was designed and constructed, yielding a range of $t^D/(t^D + t^A)$ values from less than 0.005 to 1.00, with cycle lengths variable from $1/8$ second to approximately 4 minutes. The subjects for these experiments were female white Carneaux pigeons about six years of age. In order to insure a constant "drive level," the birds were maintained throughout the experiment at 80 per cent (± 15 gm.) of their weight under ad libitum feeding. The pigeons were trained to peck at a key, or trans-illuminated disk, at one end of the experimental cage; during preliminary training, each key-pecking response was reinforced by the presentation for 3 seconds of a small grain hopper several inches beneath the key. For a more detailed description of the experimental cage and conditioning technique see the account of these procedures given by Ferster.⁵ Response data were taken in the form of cumulative response curves, total number of responses and reinforcements per session, and polygraph records of response distributions in time.

Two types of studies from which the present data are taken are progressing in this laboratory. These data will be presented in greater detail in subsequent publications. In one, $t^D/(t^D + t^A)$ is maintained at a constant value while cycle length is systematically varied. In the second, part of a doctoral dissertation by one of the authors, the length of t^D is varied while total cycle length is maintained constant. On each schedule studied, the original three restrictions apply: t^D and t^A are held constant, t^D and t^A are alternated, and only the first response in t^D is reinforced. After initial training of the key-pecking response, all birds were placed on a schedule permitting the first response in any 30-second period to be reinforced ($t^D/(t^D + t^A) = 1.00$, $t^D + t^A = 30$ seconds), a schedule identical with a 30-second "fixed interval."

After approximately fourteen days of preliminary training on the above intermittent schedule, birds in the first experiment were shifted to a new schedule, in which the total cycle length was 30 seconds and t^D was 1.5 seconds ($t^D/(t^D + t^A) = 0.05$). $t^D/(t^D + t^A)$ was held constant at 0.05 throughout this study. Data are being taken on cycle lengths of 30, 15, 7.5, 3.75, 1.88, and 0.94 seconds. Each bird is kept on a given schedule until its response rate has reached a steady state, as defined by the following stability criterion. The first seven days on any schedule are not considered in computing stability. For the next six days the mean of the first three days of the six is compared with that of the last three days; if the difference between these means is less than 5 per cent of the six days' mean, the bird is considered to have stabilized and is shifted to the next schedule. If the difference between submeans is greater than 5 per cent of the grand mean, another experimental day is added and similar calculations are made for that day and the five immediately preceding it. Such extensions of the experiment and calculations of stability are continued daily until the bird reaches the afore-mentioned 5 per cent criterion. Though each bird spends 20 minutes daily, weight permitting, in the experimental box, only the last 15 minutes are employed in calculating stability criteria.

Figure 2 displays typical cumulative response curves of bird No. 8 for complete 20-minute sessions late in training on each schedule with cycle lengths of 30, 15, 7.5, and 3.75 seconds and respective t^D 's of 1.5, 0.75, 0.38, and 0.19 seconds. Curves are not shown for cycle lengths of 1.88 and 0.94 seconds, since these, as it turned out, would be superimposed on the 3.75-second curve. Four birds are being used in this study, and all have exhibited consistent functions. With long cycle lengths the rate of responding is relatively low; as the cycle length decreases, rate of response increases, appearing to approach an asymptote at about 3.75 seconds. "Fixed interval" responding is displayed when the total cycle length is 30 seconds. As the cycle is shortened to approximately 2 seconds, however, the bird's response output develops features characteristic of "random ratio" behavior and approaches a mean value of 20 responses per reinforcement.

The second current investigation involves an examination of the effects of decreasing $t^D/(t^D + t^A)$ from 1.00 through 0.35, 0.05, 0.03, 0.02, and 0.013, while maintaining cycle length constant at 30 seconds. Each bird is used daily, weight permitting, for a total of 30 minutes. Figure 3 indicates representative cumulative response curves of bird No. 3 on the enumerated $t^D/(t^D + t^A)$ values. When $t^D/(t^D + t^A)$ equals 1.00, the cumulative curve is marked by temporal discrimina-

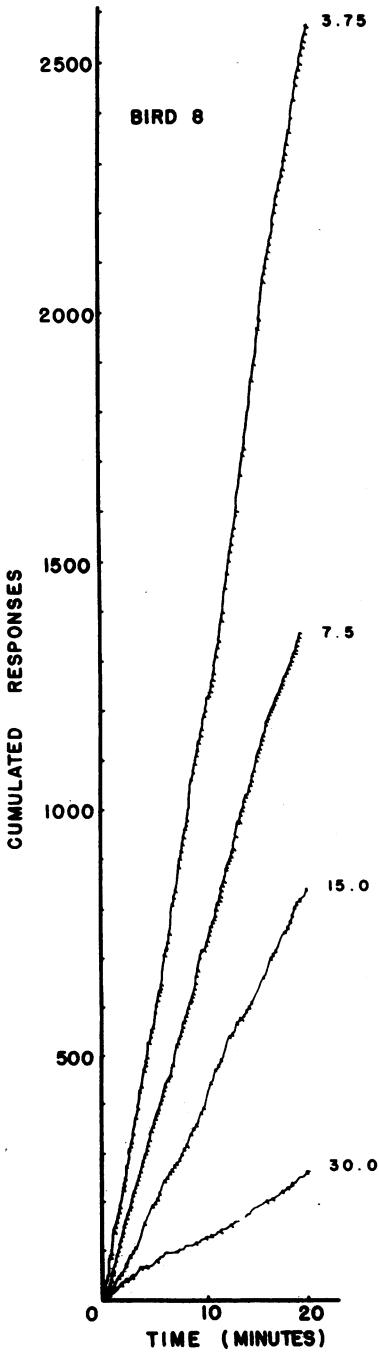


FIG. 2.—Typical cumulative response curves for bird No. 8. The value of $t^D + t^A$ is shown at the end of each curve. In all cases $t^D/(t^D + t^A) = 0.05$. Reinforcements are indicated by short diagonal lines.

tions typical of responding under "fixed interval" schedules. As the t^D period shortens, the response rates increase and the general shape of the curves becomes more similar to that under "fixed ratio" or high mean "variable ratio" schedules.

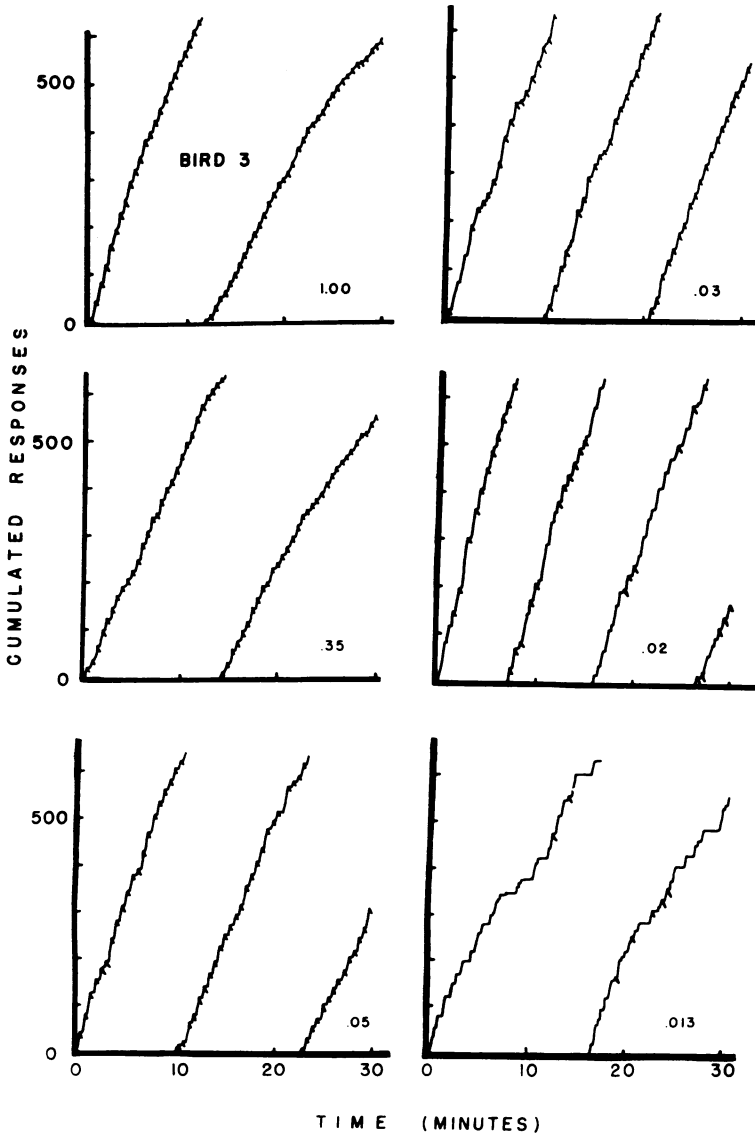


FIG. 3.—Typical cumulative response curves for bird No. 3. In order to conserve space, the curves are displaced to the base line after approximately 650 responses have accumulated. The value of $t^D / (t^D + t^\Delta)$ is shown for each curve. In all cases $t^D + t^\Delta = 30$ seconds. Reinforcements are indicated by short diagonal lines.

"Breaks" after reinforcements appear to increase with a decrease in t^D , a finding already known to be the outcome of successive increases in the magnitude of the fixed ratio.⁶ The last curve ($t^D / (t^D + t^\Delta) = 0.013$) resembles that obtained by

Skinner² after shifting subjects from one "fixed ratio" to a much higher "fixed ratio." Three other birds employed in this second study show similar changes from "fixed interval"-like response curves to "ratio"-like curves and an increase in response rate as $t^D/(t^D + t^A)$ decreases. With continued shortening of t^D , perhaps beyond values used in this study, extinction of the response would eventually occur, since the probability of coincidence between response and t^D would fall so low (at any actually attainable rate of response) as to make the maintenance of behavior impossible.

SUMMARY

An attempt was made to show how operant reinforcement schedules may be viewed as special cases of one general conceptual framework employing certain temporal parameters as its major independent variables. Some preliminary data were presented to show how shifts from "interval" to "ratio" response characteristics can be obtained by varying these temporal parameters.

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¹ B. F. Skinner, *Science and Human Behavior* (New York: Macmillan Co., 1953), p. 100.

² B. F. Skinner, *The Behavior of Organisms* (New York: D. Appleton-Century Co., 1938).

³ Skinner, *Science and Human Behavior*, p. 105. See also A. V. Lagmay, "A Pacing Technique in the Study of Some Schedules of Reinforcement" (unpublished Doctor's dissertation, Harvard University, 1955), and E. B. Newman and D. Anger, "The Effect upon Simple Animal Behavior of Different Frequencies of Reinforcement" (United States Army, Office of the Surgeon General, Final Report DA-49-007-MD-408, 1954).

⁴ C. M. Brandauer, unpublished data.

⁵ C. B. Ferster, "The Use of the Free Operant in the Analysis of Behavior," *Psychol. Bull.*, **50**, 263-274, 1953.

⁶ J. J. Boren, "Response Rate and Resistance to Extinction as Functions of the Fixed Ratio" (unpublished Doctor's dissertation, Columbia University, 1953), and Skinner, *The Behavior of Organisms*.