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*EFFECT OF THE RIGIDITY OF THE INNER CORE ON THE FUNDAMENTAL OSCILLATION OF THE EARTH**

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The Bullen B and Gutenberg models of the earth which we studied previously gave periods for the spheroidal oscillation $n = 2$ of 53.70 and 53.52 min respectively, as against the average observed seismic and gravimetric values of 53.9 min. In order to explain this discrepancy, we have studied the effect on the period of an assumed rigidity in the inner core ($r < 1{,}250$ km). It is found that the period of the "core" oscillation of about 101 min diminishes rapidly with increasing rigidity μ of the inner core, reaching an asymptotic value of about 53.8 min at large μ, while simultaneously the amplitude spreads into the mantle, eventually assuming the pattern of a normal oscillation at the asymptotic period. The observed period of 53.9 min is reached at a value of μ of about $^1/_2 \times 10^{12}$ dyne/cm^2, and could be fitted within the observational error into the range of 1.5×10^{12} to 4×10^{12} for μ inferred by Bullen on the basis of seismic data.

1. *Introduction.*—In the interpretation of the spectrum of the earth which was observed gravimetrically[1] and seismically[2] on the records of the great Chilean earthquake of May 22, 1960, we compared the observed periods with theoretical ones evaluated for Bullen's model B and Gutenberg's model of the earth.[3] For the fundamental spheroidal oscillation $n = 2$, these periods came out 53.70 and 53.52 respectively, compared with the average of 53.89 min for the observed gravimetric[1] doublet of 54.98 and 52.80, and 53.9 min for the center of the observed seismic[2] doublet of 54.7 and 53.1 min. The discrepancy of 0.2 min between theory and observation for Bullen's model and of about 0.4 min for Gutenberg's model is noteworthy in view of the better agreement between theoretical and observed periods that was found for the higher modes, especially for the Gutenberg model, and the

quantitative agreement with theory[4–6] shown by the observed rotational splitting of the $n = 2$ mode.

Since the amplitude of the higher modes, whose periods are well accounted for by theory, is confined principally to the mantle, we must look for possible modifications of the model in the core, which in the past was assumed to be liquid throughout. Now a body of evidence has been accumulating since 1936[7] and continuing to date,[8–10] pointing to a possible rigidity in the first 1,250 km from the center in the core. In a recent summary of the evidence, K. E. Bullen[11] infers that the rigidity of the inner core is probably not less than 1.5×10^{12} dyne/cm^2, and may be as high as 4×10^{12} dyne/cm^2, though direct identification of either reflection from or transmission through the inner core of *shear* waves is still lacking.[12]

We have, accordingly, undertaken to investigate the possible effects of a small rigidity in the inner core on the periods of free oscillation of the earth. It was clear at the outset that rigidity would tend to *decrease* the period rather than bring about the necessary increase from 53.5 or 53.7 min, computed on the assumption of a liquid core, to the observed value of about 53.9 min. Actually, we obtain a period of 53.9 min at a value of μ of about $^1/_2 \times 10^{12}$ dyne/cm^2, but this oscillation has its origin in the "core" mode, which for zero rigidity in the inner core has a period of 100.9 min[13] for Bullen's model B. At $\mu = 0$, the amplitude of the "core" mode is confined entirely to the core, as is shown in Figure 1*a*. At $\mu = {}^1/_{16}$ (in units of 10^{12} dyne/cm^2), the period is already down to 71.9 min, and some energy begins to leak into the mantle, as shown in Figure 1*b*. At $\mu = {}^3/_{16}$ shown in Figure 1*d*, the amplitude has assumed a normal distribution penetrating fully into the mantle like the fundamental oscillation shown in Figure 1*e*, except that there is now a nodal point at $r = 0.28$.

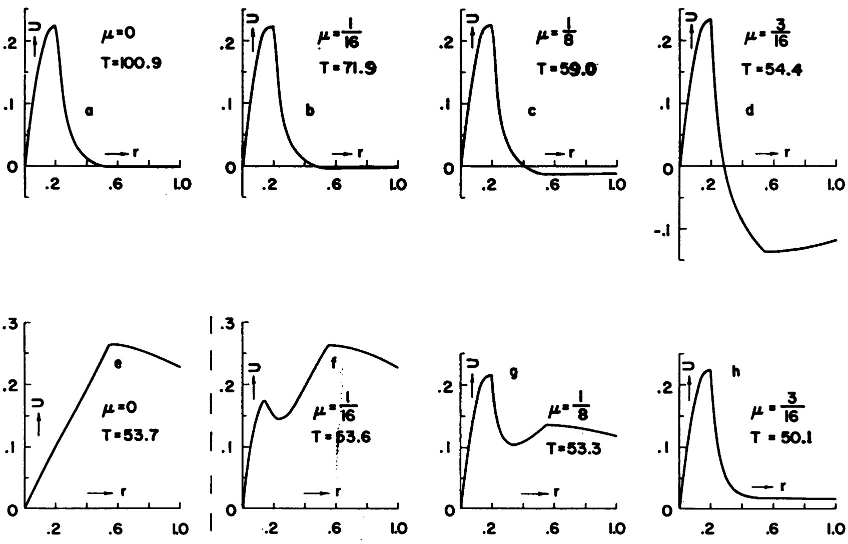

Fig. 1.—Transformation of the core oscillation into a regular one (*a*, *b*, *c*, *d*) and vice versa (*e*, *f*, *g*, *h*) for a spheroidal oscillation $n = 2$. μ denotes the rigidity of the inner core in units of 10^{12} dyne/cm^2, and $r = 1$ is the surface of the earth. T is the period in min. U is the radial displacement. The radius of the inner core was taken as 1,250 km, and the model assumed is Bullen B.

2. *Coupling of the Core Modes with the Regular Modes.*—The "core" oscillation of 100.9 min can also be obtained directly by disregarding the mantle altogether and imposing at the core-mantle boundary $r = b$ the conditions

$$U = 0, \qquad \frac{\partial \psi_n}{\partial r} + \frac{(n+1)}{b}\psi_n = 0, \tag{1}$$

where U denotes the radial displacement and ψ_n the perturbation in the gravitational potential. For the liquid core of Bullen model B, we get a period of 101 min, in line with the very small amplitudes in the mantle which were found[13] for this oscillation. The results presented below were obtained, however, by solving the whole system of equations in the core as well as in the mantle. Upon introduction of rigidity into the central core, the frequency σ increases rapidly with μ as shown by line 1 in Figure 2. Lines 2 and 3 show the trend of the overtones of core oscillations. The frequency of the regular oscillation increases less rapidly with μ, as is shown by curve 4 and its overtone curve 5. A point of intersection such as $A(\mu_1, \sigma_1)$ would mean that in a model of inner core rigidity μ_1 there is a core oscillation and a regular oscillation of equal frequency σ_1. Actually, such points of intersection have not been found. What happens is that the curves separate and interchange slopes in the manner shown in Figure 3. The eigenvectors on branch 1′ are similar to vectors of the 101 min core oscillation for a liquid core. The amplitude in the mantle is negligible for $\mu < \mu_1$, as is shown in Figures 1*a* and 1*b*. Near $\mu \simeq \mu_1$ (Fig. 1*c*), the amplitude in the mantle begins to become appreciable, and for $\mu > \mu_1$ (Fig. 1*d*), both mantle and core participate in the oscillation.

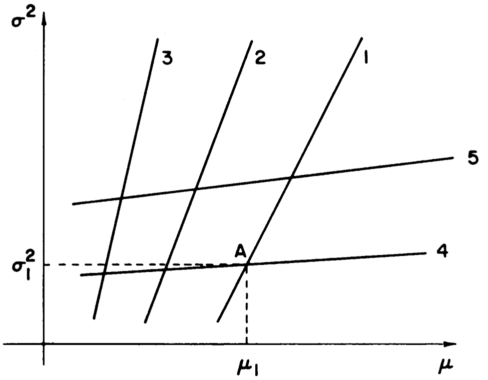

FIG. 2.—The square of the frequency σ^2 as a function of the rigidity μ of the inner core. Curve 1 is the core oscillation; 2 and 3 are overtones. The regular oscillations are shown by curves 4 and 5.

The reverse happens on the curve 4′–1″. The oscillations on the 4′ branch are of the regular type, as shown by Figure 1*f*. Near $\mu \simeq \mu_1$, the amplitude in the mantle begins to decrease relative to the core (Fig. 1*g*), and on the branch 1″ the amplitude withdraws from the mantle into the core, as shown by Figure 1*h*.

3. *Discussion of Results.*—The pattern of the spectra of the core oscillations and regular ones as illustrated in Figure 2 is complicated and is being investigated fur-

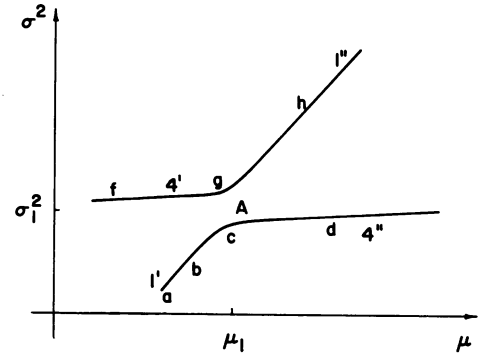

FIG. 3.—Transformation between the core and regular oscillations near a branching point A of Figure 2.

ther. The transformed core oscillation shown in the upper part of Figure 1 gives periods of 53.87 min at $\mu = {}^1/_2 \times 10^{12}$ and 53.83 min at $\mu = 2 \times 10^{12}$. The observed period of 53.9 min for the spheroidal oscillation $n = 2$ could thus be accounted for by assuming a rigidity in the first 1,250 km of the core of the order of ${}^1/_2 \times 10^{12}$ dyne/cm.^{2} Because of the slow variation of the period with rigidity, Bullen's inferred values for μ of 1.5 to 4 $\times$ 10^{12} would give a period still within the observational error.[14]

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