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Model of clustering of earthquakes

(earthquake prediction/crack propagation/cohesion/dynamical friction)

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ABSTRACT A two-dimensional model of the expansion of a crack in an elastic medium is considered in which friction depends on the slip rate and the modulus of cohesion depends on the speed of expansion of the crack. Elastic waves are neglected (quasi-static model). Under some conditions, the expansion of the crack is realized by the alternation of slow and fast episodes ("shocks") of slip. This offers a possible qualitative explanation of several forms of earthquake clustering, including clustering that is premonitory to strong earthquakes.

The clustering of earthquakes in space–time is an important feature of earthquake sequences on both global and regional scales. Most earthquakes, except possibly the strongest ones, come in clusters. Most clusters consist of a mian shock and its aftershocks. However, other types of clusters are also known—foreshocks, swarms (i.e., clusters of earthquakes of similar energy), and multiple earthquakes clustered so densely that they are considered as a single event.

The clustering of earthquakes is referred to below merely as "clustering", and the cluster of earthquakes is referred to as the "cluster". Phenomenological and statistical models of clustering have attracted much attention (1, 2), which increased recently when it was noticed that unusually large clusters often precede strong earthquakes (3–9). However, the mechanism of clustering is not clear and presents an important problem due to its intrinsic connection with one of the unexplained fundamental features of earthquakes—their interaction with each other over wide distances and time intervals. A model for clustering is suggested in this paper. The expansion of a two-dimensional crack in an elastic medium is considered; the dynamic friction k is a function of the slip rate $\dot{w}$ and the modulus of cohesion K is a function of the rate of expansion of the crack $\dot{l}$. The assumed shapes of these functions are shown in Fig. 1 *A* and *B*; the reasons for the assumptions have been discussed in refs. 10 and 11.

It will be shown in this paper that the expansion of a crack in such a model is realized under some conditions through the alternation of two phases (Fig. 1*C*). During the first phase, the slip rate is larger and the rate of expansion is smaller than during the second phase. It seems natural to associate the first phase with an earthquake and the second phase with a creep event or with a slow earthquake (12). We will call the first phase a "shock" and the second phase a "slow phase."

This sequence of shocks can then be associated with the clustering of earthquakes. Obviously, the suggested model is only a remote approximation to the actual earthquake source. It describes only one element, although, as it happens, a consequential element, of the earthquake mechanism: the influence of dynamic friction and dynamic cohesion on the expansion of a *single* crack. This model has several limitations. (*i*) It is two dimensional, so that, in three dimensions, it may only describe the growth in width of an elongated crack. (*ii*) The influence of elastic waves is neglected. (*iii*) We consider only a single crack although the interaction among different cracks is obviously intrinsic to clustering.

We assume, however, that the alternation of shocks and slow phases in our model is qualitatively applicable to reality. Under this assumption, another feature of the model deserves attention: the number of alternations increases significantly for the case in which the tip of the expanding crack enters the local maximum of stress or the local minimum of friction. In other words, in such cases, the model shows especially large clusters of shocks. This offers a qualitative explanation of some observed premonitory seismicity patterns.

THE PROBLEM

We consider a homogeneous elastic plane that has Lamé parameters λ and μ. Let $u(x,y)$, $v(x,y)$ be the components of the displacement along the rectangular coordinates (x,y). The crack is the segment $y = 0$, $-l \leq x \leq l$, on which the displacement

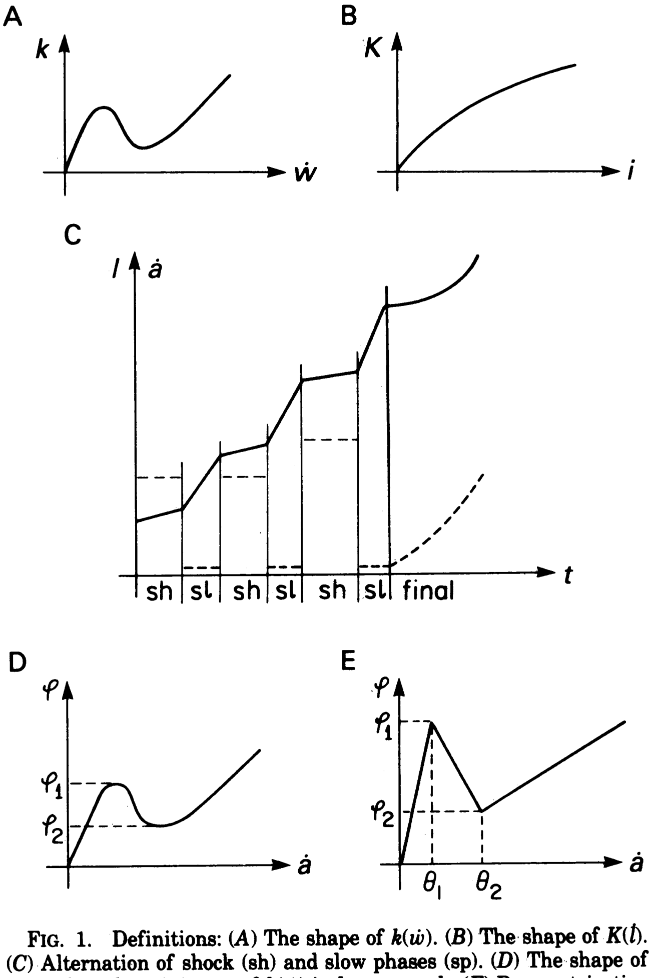

FIG. 1. Definitions: (*A*) The shape of $k(\dot{w})$. (*B*) The shape of $K(\dot{l})$. (*C*) Alternation of shock (sh) and slow phases (sp). (*D*) The shape of $\phi(\dot{a})$ when the minimum of $k(\dot{w})$ is deep enough. (*E*) Parametrization of $\phi(\dot{a})$.

$u(x,y)$ may be discontinuous. Let X_x, Y_y, X_y be the components of the stress in a common notation.

At infinity,

$$X_x = Y_y = q, \qquad X_y = \tau, \tag{1}$$

where τ and q are constants.

Let us ascribe the indexes "+" and "−" to the functions on the opposite sides of the crack: $f^{\pm} = \lim_{\varepsilon \downarrow 0} f/y = \pm\varepsilon$. The boundary conditions on the sides of the crack are the following:

$$\begin{aligned} X_y^+ &= X_y^- = g\,(x) \\ Y_y^+ &= Y_y^- \\ v^+ &= v^-. \end{aligned} \tag{2}$$

Here

$$g(x) = qk[\dot{w}(t,x)] + G(t,x), \tag{3}$$

$w = u^+ - u^-$, k is the friction coefficient, G is the cohesive force, and dots above symbol mean differentiation with respect to time.

According to ref. 10, $G(t)$ is different from 0 only near the tip of the crack, and

$$\int_{-l}^{l} \frac{G(t,x)dx}{(l^2 - x^2)^{1/2}} = \mathrm{K}(\dot{l}). \tag{4}$$

Let us first assume that $G(t,x)$ is known. Then, by using the method of complex potentials (13) we find the following relationship between the stress at infinity (q,τ), the stress at the crack $g(x)$, and the relative displacement of the sides of the crack (13):

$$-\tau + g(x,t) = \frac{\gamma}{\pi} \int_{-l}^{l} \frac{dw(t,y)}{dy} \frac{dy}{y - x}, \tag{5}$$

where $\gamma = \mu(\lambda + \mu)/(\lambda + 2\mu)$. According to ref. 10,

$$\left(\frac{l}{2}\right)^{1/2} \int_{-l}^{+l} \frac{\tau - qk[\dot{w}(t,x)]}{(l^2 - x^2)^{1/2}}\, dx = K(\dot{l}). \tag{6}$$

The boundary and initial conditions are

$$\begin{aligned} w(0,x) &= w_0(x) \\ l(0) &= l_0 \\ w(t,x) &= 0\ (x = \pm l(t),\ t \geq 0). \end{aligned} \tag{7}$$

Our problem is to find $w(t,x)$ and $l(t)$ from Eqs. 5–7, assuming that $G(t,x)$, $k(\dot{w})$, $w_0(x)$, and l_0 are known.

Approximate Solution. We assume the following first Galerkin approximation:

$$\dot{w}(t,x) = \dot{a}(t) \left(1 - \frac{x^2}{l^2}\right)^{1/2} \tag{8}$$

This is the exact solution in the case where $k = 0$, $G = 0$. For the new unknown function $a(t)$, we have $a(0) = 0$. Eqs. 5–7 can then be reduced to the following:

$$\tau - q\phi(\dot{a}) = \gamma \frac{a}{l} \tag{9a}$$

$$a = \left(\frac{(2)^{1/2}}{\pi}\right)^{1/2} \cdot \frac{1}{\gamma} K(\dot{l})\,(l)^{1/2}, \frac{\mathrm{k}[\lambda(1 - x^2)]^{1/2}}{(1 - x^2)^{1/2}}\, dx \tag{9b}$$

where

$$\phi(\lambda) = \frac{1}{\pi} \int_{-1}^{+1} \frac{\mathrm{k}[\lambda(1 - x^2)^{1/2}]}{(1 - x^2)^{1/2}}\, dx \tag{10}$$

Qualitative Analysis of $l(t)$, $a(t)$. In the most general case, $\phi(\dot{a})$ has two extrema similar to $k(\dot{w})$; this will take place when the minimum of $k(\dot{w})$ is deep enough. Let us introduce the notation shown in Fig. 1 *D* and *E*. The shape of the phase trajectories (a,l) depends on the relationship between τ/q and the extrema of $\phi(\dot{a})$. Three possible cases are shown in Fig. 2.

For the case $\tau/q < \phi_2$, only the left branch of $\phi(\dot{a})$, below the level $\phi = \phi_2$, can be reached. For the case $\phi_2 < \tau/q < \phi_1$, there may be a switch between the two ascending branches of $\phi(\dot{a})$ and $\dot{l}$ will have one discontinuity. The case $\tau/q > \phi_1$ is the most interesting for the study of clustering.

In this case, Eq. 9, is compatible with three different phase fields, and multiple switching from one field to another is possible at their boundaries. This is the case referred to in the *Introduction*.

In all three cases, there is a similar *final stage*: At sufficiently large t, $l(t)$ increases smoothly with increasing speed; this indicates that the limit of applicability of our model is approached. In our approximation, for a generalization to x-dependent friction, cohesion, and shear stress, it is sufficient to introduce the following substitutions in Eq. 9: Replace $k(\dot{w})$ by $k(\dot{w})f(x)$, $K(\dot{l})$ by $K(\dot{l})d(x)$, and τ by

$$\int_{-l}^{l} \frac{\tau(x)dx}{(l^2 - x^2)^{1/2}}.$$

Numerical Experiment. Let N be the number of shocks that occurs during the expansion of the crack in the time interval between the time it originates ($t = 0$, $l = l_0$) and the final phase.

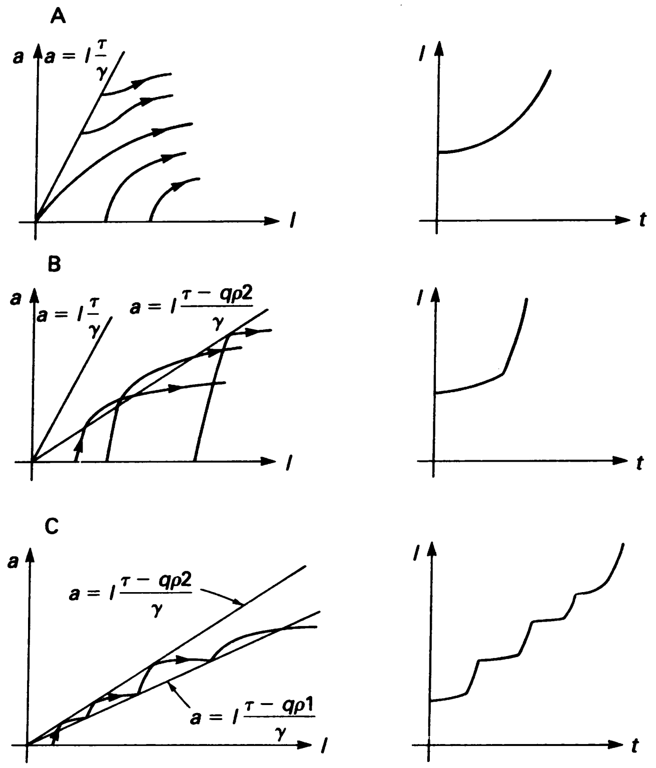

FIG. 2. Shape of the phase trajectories and $l(t)$. (*A*) $\tau/q < \phi_2$. (*B*) $\phi_2 < \tau/q < \phi_1$. (*C*) $\tau/q > \phi_1$.

Table 1. Parameter ranges

Parameter	Range	Parameter	Range
K_0	$0.2\text{–}100 \times 10^8$	Θ_1	100–500
τ	$0.05\text{–}5 \times 10^8$	Θ_2	0.01–0.05
q	$1\text{–}10 \times 10^8$	v	1.5–3
ϕ_1	0.05–3	l_0	$1\text{–}1.5 \times 10^4$
ϕ_2	0.05–3	μ	5×10^{11}

Values are in CGSE units or dimensionless.

The influence of shear stress, cohesion, and friction on N was studied numerically. In Fig. 2, N is the number of such segments that go from $a = (1/\gamma)(\tau - q\phi_1)l$ to $a = (1/\gamma)(\tau - q\phi_2)l$.

The large values of N correspond in our model to a cluster within the limitations discussed above. The functions $K(\dot{l})$ and $\tau(x)$ were parametrized as follows:

$$K = K_0 \dot{l}^{1/v}$$

$$\tau(x) = c_1 + c_2[(x - c_3)^2 + c_4]^{-1/2} + c_5[(x - c_6)^2 + c_7]^{-1/2}.$$

This parametrization is the simplest way to generate extrema on the x axis. $f(x)$ was parametrized in the same way as $\tau(x)$.

The function $\phi(\dot{a})$ was parametrized as shown in Fig. 1*E*. In this parametrization, Eq. **9 a** and **b** becomes

$$\phi(\dot{a}) = \frac{1}{q}\left(\tau - \gamma\frac{a}{l}\right) \quad \text{[11a]}$$

$$\dot{l} = \left[\frac{\pi}{(2)^{1/2}\gamma}\cdot\frac{a}{K_0(l)^{1/2}}\right]^v \quad \text{[11b]}$$

We computed $l(t)$, $a(t)$, and N for the numerous parameter combinations within the limits given in Table 1. The local extrema of $\tau(x)$ were considered within the limits given in Table 2. An extremum in the friction on the x axis was introduced with similar range of c_1, c_2, c_3, and $c_5 = 0$. According to Eq. **9a**, it is qualitatively equivalent to an opposite extremum in $\tau(x)$. Our computations actually cover a wider range of parameters due to the similarity, which follows from Eq. 11. Thus, Eq. 11 will not change under the following substitutions: $q \to q' \times 10^{\alpha}$; $\tau \to \tau' \times 10^{\alpha}$; $l \to l' \times 10^{2\beta}$; $\mu \to \mu' \times 10^{\alpha+2\beta}$; $K_0 \to K_0' \times 10^{\alpha+\beta-2\beta/v}$; $a \to a'$; $t \to t'$. Let us now discuss the results of the computations. As described above, the clusters may appear in our model only if the shear stress is large enough: $\tau > q\phi\varepsilon_1$. The computations for such τ have shown that an increase in N may be caused by any of three factors: (*i*) friction, similar to a "dry" one (large Θ_1); (*ii*) large cohesion (large K_0); or (*iii*) the presence of a local maximum of shear stress or a local minimum of friction.

The values of N for a shear stress independent of x (τ = constant) ranged between 0 and 110; with a local maximum in $\tau(x)$, N reached 150. The large values of N are unstable to variations in the parameters; in other words, the maxima of N are rather narrow. We did not look for an absolute maximum.

DISCUSSION

The formation of the clusters of shocks in our model can be qualitatively explained by the shape of functions $k(\dot{w})$ and $K(\dot{l})$, which represent friction and cohesion. At the moment of initial rupture, $\dot{w}$ is relatively small, the friction offers small resistance to the slip, and the slip accelerates. This leads to an increase in the friction and the slip accordingly slows down. This in turn leads to a decrease in the friction and so on. The expansion of the crack performs a self-induced acceleration and deceleration in a similar way because cohesion increases with the increase in $\dot{l}$. It is less obvious why $\dot{a}$ and $\dot{l}$ do not increase simultaneously but in turn. The increase in clustering due to the three factors listed above also has a feasible qualitative explanation.

Larger Θ_1 means the steeper increase of friction with the increase of $\dot{a}$; it will make the switching between shocks and slow phases quicker. The larger K_0 will cause a similar effect. A local minimum in friction or a maximum in shear stress will facilitate the beginning of the shock, while deceleration will be introduced on the other segments of the crack.

The assumed shapes of the friction $k(\dot{w})$ and cohesion $K(\dot{l})$ are rather specific. They may be questioned for rocks, as corresponding laboratory experiments are not numerous. However, our reservations are to some extent diminished by the fact that these shapes were not introduced *a posteriori*, to explain the clustering of earthquakes, but quite independently on seismological grounds. At least two features of our model seem to disagree with the phenomenology of earthquake sequences. One feature concerns the energy released by consecutive shocks. In the absence of friction and cohesion, the energy released by a single shock would be proportional to the increment of the product (τla) (14, 15). Let us denote this increment δ_i, $i = 1, 2\dots$ being the sequence number of the shock; the time of the beginning of the ith shock we denote as t_i. In the largest part of the considered parameter space, δ_i increases and t_i decreases with the increase of i. For exceptional combinations of the parameters, δ_i will have a local minimum on some interval of i. If δ_i is actually a measure of the released energy, then the foreshocks occur under much wider conditions than the aftershocks. This is contrary to the observed phenomenology and may be connected with a second inadequate feature of our model—namely, the absence of a mechanism to arrest the final stage, the infinitely accelerating expansion of the crack. Our model shows a monotonic increase in the energy of the foreshocks δ_i simultaneous with a decrease in the time interval ($t_i - t_{i-1}$) between them. This again is not always observed in actual sequences of foreshocks. It seems, therefore, that the seismological interpretation of the details of our model is hardly warranted. This model may be considered only as a possible explanation of the basic phenomenon of clustering. This explanation is supported by the fact that it follows from properties of the friction and cohesion formulated *a priori*, independent of the data on earthquake sequences.

This explanation seems qualitatively relevant to multiple earthquakes (16), silent earthquakes (12), swarms (3, 4, 7, 9) and bursts of seismicity premonitory to strong earthquakes (6). Our explanation is by no means unique. For example, models that have barriers (17) and asperities (5) may also explain the above-mentioned phenomena. The useful feature of our model is that it may explain why some premonitory clusters appear at large distances from the subsequent strong earthquake. One possible explanation is connected with the conditions for the occurrence of large clusters in our model: the shear-stress is large enough ($\tau > q\phi_1$) and there are local concentrations of shear stress or local minima of friction or both. It seems natural to expect that these conditions arise in the whole area affected by the factors that trigger a strong earthquake. There are no reasons to expect that this area is confined by the immediate vicinity of a strong earthquake. For example, many precursors were observed at

Table 2. Extremum ranges

Extremum	Range	Extremum	Range
c_1	$1\text{–}5 \times 10^8$	c_5	$-1\text{–}1 \times 10^8$
c_2	$-0.5\text{–}20 \times 10^8$	c_6	$0\text{–}10 \times 10^8$
c_3	$1\text{–}4 \times 10^3$	c_7	0–1
c_4	0.2–3		

distances up to $10^{0.43M}$ from the epicenter of a coming earthquake of magnitude M (18). The increase in stress and local stress concentrations need no comment. Decreases in friction can arise when the fault zone is penetrated by a fluid, which erodes the strength due to the Rebinder effect (19), lubricates the fault, or causes an increase in pore pressure.

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