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Locomotion: Dealing with friction

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Contributed by V. Radhakrishnan, February 10, 1998

ABSTRACT To move on land, in water, or in the air, even at constant speed and at the same level, always requires an expenditure of energy. The resistance to motion that has to be overcome is of many different kinds depending on size, speed, and the characteristics of the medium, and is a fascinating subject in itself. Even more interesting are nature's stratagems and solutions toward minimizing the effort involved in the locomotion of different types of living creatures, and humans' imitations and inventions in an attempt to do at least as well.

The Imperative to Move

The world of animals—which includes all of us—is characterized by voluntary motion. It is necessary to move to find food or a mate, or to exercise the muscles needed to move.

For reasons that this paper is all about, expenditure of energy always seems to be required to move, whether on land, in water, or in the air. It is only in the space between the stars that bodies—such as the members of the solar system—are able to move for eons with little or no expenditure of the energy associated with their motions. We attribute this seemingly everlasting persistence of motion—the first of Newton's laws—to the absence of material in space, which the bodies otherwise would rub against, creating heat at the expense of their kinetic energy. For terrestrial objects, both living and inanimate, one is in continual contact with one or more of the solid, liquid, and gaseous parts of our environment, resulting in an expenditure of energy for motion even at uniform speed and at the same level.

Reducing the resistance that has to be overcome, which I shall call friction, allows a greater range of travel for a given input of energy, or a greater speed for a given input of power. In the course of evolution and adaptation, one absolute requirement for survival was surely that the energy needed to move to the location of the next meal should be less on average than that acquired from consuming the last one. Similarly, that the speed required either to capture prey, or to avoid becoming one, should be achievable with the power that can be summoned at short notice from the muscles.

In humans' attempts to build vehicles capable of greater range and speed on land, in water, and in the air, nature has provided much inspiration and numerous examples, and in my attempts to understand the principles behind the functioning of craft in these different media, I have come to learn about and marvel at nature's creations.

Getting Around Friction

Civilization and its manifestations often require the moving of heavy objects and undoubtedly provided the earliest opportunities for invention of devices to reduce the effort involved. The wheel is considered the greatest such invention and the three power sources used in the Middle Ages—animal, water, and wind—all were exploited by means of wheels. I presume that the earliest form in which the wheel appeared must have been logs on which heavy stones were rolled such as those of which the pyramids were built. The force, or thrust, required to move the stone is reduced from the colossal sliding friction over the ground, which it must have, to the rolling friction of the stone over the logs and that of the logs over the ground, which together still could be provided by a finite number of slaves, and presumably some form of persuasion (Fig. 1).

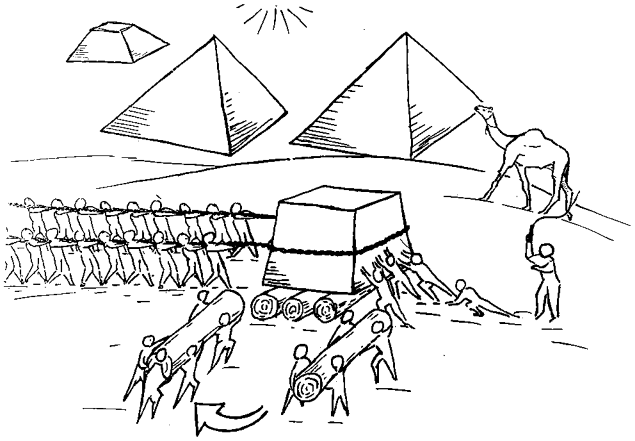

FIG. 1. Using rollers to move heavy stones as was presumably done when building the pyramids. As each roller comes out at the rear it has to be reintroduced in front, a tricky operation that must have added not a little to the agony of the slaves.

The introduction of a simple axle in the center as in a chariot or wagon wheel was a giant leap forward, but at the expense of reintroducing sliding friction between the axle and the part of the wheel on which it rested (Fig. 2). Although the full load was still on this common surface, the distance it had to be dragged was reduced by the large ratio of the diameters, and this area could be lubricated. Lubricants magically reduce sliding friction between solids and find universal application in places where they can be contained, e.g., between axle and hub. An extraordinary example of self-lubrication, and the only case I know where sliding friction can compete with rolling friction, is that of runners on ice. They enable skaters and ice yachts to reach impressively high speeds for the input powers involved.

The next ingenious step in the evolution of the wheel was to make it all rolling friction by putting back the cylindrical rollers, or equivalently spherical balls, in an annular space between the axle and the hub of the wheel, where they can remain in place. This idea was discussed in detail and illustrated in drawings by

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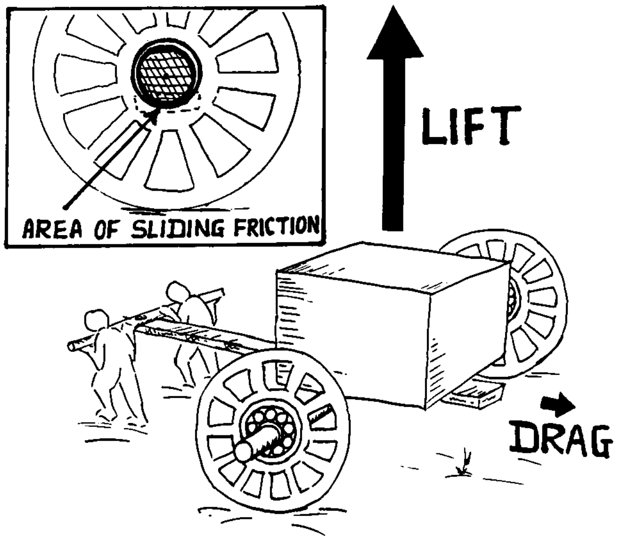

FIG. 2. The wheel and axle, among the greatest inventions ever. (*Inset*) Sliding friction has to be overcome at one place, but which can now be easily lubricated. The ultimate form of the wheel. Rollers (or balls) between the axle and the wheel hub make it all rolling friction. The ratio of lift to drag could easily be as high as 100:1.

Leonardo da Vinci, who also invented the (now standard) cage that prevents the balls and rollers from rubbing against each other. It is such a device on which all objects that have to be moved on terra firma are ideally mounted, and which provides a support for the weight that I shall call "lift" and a very small resistance to horizontal motion that I shall call "drag," the ratio between the two being very much greater than unity at low speeds (Fig. 2). For movement along fixed routes an improvement that revolutionized the transport of goods and people over large distances was the introduction of railways. The rolling friction of steel on steel was so much less than other wheels on other surfaces that it made a dramatic change in transportation patterns over land, which endures to this day.

Needing and Using Friction

Let me dispel any growing feeling that friction is all bad by pointing out that the slaves in my illustration (Fig. 1) would not have been able to generate any force to move the stone, but for the enormous friction between their feet and the ground. A lubricating film of oil under a heavy slab would facilitate pushing it over the floor, but the same oil under one's foot is unlikely to help in the pushing. We shall encounter later the more subtle manifestations of this essential role of friction in the generation of forces in fluid media, but let us just note here that one always needs friction to generate the force to overcome the drag caused by friction elsewhere. Optimization then strangely becomes the exercise of minimizing friction at one end of the system, while maximizing it at the other.

In the case of self-propelled land vehicles, we have the even stranger case of having to satisfy both requirements at one and the same place, namely where the wheels rest on the road surface. Ideally, we need the impossible combination of maximum sliding friction required for acceleration, deceleration, and turning, together with minimum rolling friction for fuel economy in cruising. On railways, which I already have praised for very low rolling friction, problems can arise when climbing grades if the sliding friction is not high enough to prevent the wheels of the locomotive from slipping. In extreme cases such as mountain railways, a rack and pinion is used, but in wet conditions on lesser grades the inelegant solution often resorted to is of throwing sand on the rails, to provide the extra bite.

Braking, which is as important a requirement as acceleration, especially in an emergency, has all to do with the ability of friction to dissipate kinetic energy to rapidly decelerate the vehicle. The ideal way of braking is to recover and reuse the kinetic energy, as can be done with flywheels in principle, or as sometimes done on electric vehicles in practice, with electromagnetic braking and the energy fed back into the power source. But in most wheeled systems like cars, trucks, bicycles, etc, the energy is irretrievably converted to heat and sadly lost.

Getting back to the generation of thrust—to overcome drag—it is quite intuitive to appreciate that if your foot did not slip when pushing, you did as well as one possibly could in the exercise. What is less intuitive is that this is so only because the earth one was pushing against is as massive as it is, compared with the rest of the bits in the system. The law of conservation of momentum is unremitting in requiring that the momentum gained in propelling oneself (or one's vehicle) forward must be balanced by the imparting of an equal and opposite amount to whatever it was that one pushed against. So although the product of mass and velocity is inviolable, the energy carried away by whatever was "kicked back" is proportional to its velocity squared and decreases impressively with increase of its mass, becoming effectively zero for objects as massive as the earth. At the other extreme is space flight with nothing to kick against, where accelerations require ejection of part of the mass of the system itself. Of more interest are the problems associated with generating thrust in a fluid like air or water. Although the tiniest ant can push against the whole earth, even a giant whale cannot push against the whole ocean.

Making Headway Afloat

The massive pillars of Stonehenge, and those of many temples in Egypt, are believed to have been transported considerable distances over water, and to this day, heavy materials and large quantities of liquid in bulk are transported both routinely and economically by barges, tankers, and container ships.

The sources of friction encountered by vessels plying in water are several and of considerable interest. Two contributions, predominant at low speeds, are skin friction and profile drag. Skin friction depends on the roughness of the surface of the hull over which the water has to slide, and the consequent drag is proportional to the wetted surface and the square of the speed. Reduction is effected by having a smoother surface, and as little of it as possible. Profile drag is caused by the deposition of energy in turbulent eddies in the wake caused by poor streamlining of the hull. Streamlining always can be improved by giving the hull a longer and slimmer tail, but only at the expense of increase of wetted area, and of drag as a consequence. Profile drag also grows as the square of the boat speed, and thus the power to overcome both of these contributions ($=$ force $\times$ speed) will increase as the cube of the speed of the vessel. This indicates straightaway that it is cheaper to go slower, as the energy required to transport a given weight over a given distance is an increasing function of speed.

In the case of craft operating on the surface of the water (like boats, as opposed to submarines), there is yet another contribution caused by wave making that is the worst of all. For motion at constant speed on the water surface, there is nothing that varies with time as seen from the boat, and the wave pattern it sets up must perforce be stationary as viewed from it. In other words, the speed of the disturbance created by the boat must always be the same as its own. But water waves are "dispersive," and the length of the wave created by the motion increases as the square of the speed. This causes the resistance caused by wave making to increase very rapidly with speed as the length of the wave approaches that of the hull, and effectively to limit the maximum possible speed of any displacement hull to that of a wave as long as itself (Fig. 3). This phenomenon, discovered by the English engineer William Froude (14) in the last century, explains why the famous transatlantic ocean liners had to be as long as they were to do

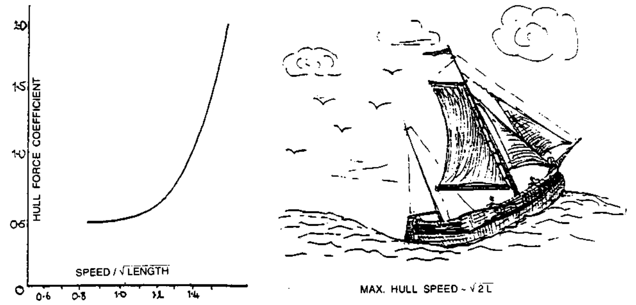

FIG. 3. Wave-making drag increases catastrophically with speed as discovered by William Froude (14). It effectively limits the speed (measured in knots) of displacement hulls to the square root of twice their waterline lengths (measured in feet).

a speed of over 30 knots. Also that is why displacement craft are effectively limited in speed to the square root of twice their water line lengths, both measured in traditional sailor's units of knots and feet.

Climbing Out or Slicing Through

Unlike the sound barrier in aeronautics, which is a speed dependent only on the temperature of the air and which has nothing to do with the properties of the aircraft trying to penetrate it, the hull speed barrier for boats is determined only by the waterline length of the particular craft. It can be beaten by using the dynamic force of the water hitting a sloping under-surface or submerged foils to violate Archimedes' principle and to lift the hull out of the water. Speed boats, planing sailboats, and water skiers all operate on this principle. Another way to get the hull out of the water is to pump air down to create a cushion underneath to support the weight. Such hovercraft, propelled by airscrews, have no drag forces associated with the water and can operate at speeds as high as 60 knots, making very effective ferries.

A different approach is to have more than one hull, the two-hull configuration known in the West as a catamaran.† Such catamarans, which have a superstructure over two slender and well-separated hulls, have great stability and can slice through the hull-speed barrier. There are more and more such craft around the world, both big and small, operating commercially at speeds well above the formal limit. The requirement of stationarity discussed earlier must continue to hold whatever the speed of the craft as long as it is constant. The only difference now is that the waves generated by it are much longer than its waterline length, and act less like a wall up which the craft has to climb (Fig. 3).

In earlier times, water craft relied heavily on wind power,‡ whose force on the sail increases as the square of the wind speed. We have just seen that skin friction and profile drag also increase as the square of the speed in water. As a result, the speed of a sailing vessel, which is not hull speed limited, would be proportional to the effective speed of the wind, and very high speeds can be achieved in very strong winds. Round-the-world nonstop voyages in a race for the Jules Verne trophy have been made by a catamaran in 74 days and by a trimaran (three-hulled) in 72 days, both craft having waterline lengths of about 90 feet. In total disregard of the hull speed formula, these multihulls have exceeded 30 knots in storm-force winds, and often averaged well over 20 knots for a full day's run, a speed rarely if ever achieved by ocean freighters, many times their length. The highest speeds however, attained under sail power (of the order of 100 knots), are by ice yachts and land yachts, on runners and wheels, respectively, both very low-friction devices.

In Thin Air

It is time to take to the air, the world of insects and birds, mad men in little flying machines, and jumbo jets with hundreds of passengers. I cannot look at any bird in flight without admiring its effortless grace, and I cannot watch any airliner taking off without feeling that it is not possible. Flight is a miracle wrought by nature and, hard to believe, successfully imitated by humans. It works because of the interplay of two very different forces called drag and lift that act at right angles to each other on any body placed in a fluid stream. Their magnitudes depend on the size, shape, and orientation (to stream direction) of the body and, of course, also on the density and velocity of the fluid. Frictional drag has contributions from skin drag and profile drag similar to those of boat hulls, with a similar dependence on the square of the speed, and with similar recipes for reduction. Unlike boats, however, an airplane is entirely immersed in its fluid, suggesting that the shapes of fish may not be bad examples for aircraft streamlining. One striking example is the use by Sir George Cayley (a pioneer in aviation experiments almost 200 years ago) of the measurements of a trout, which another pioneer, Theodore von Karman, pointed out nearly 150 years later (1), as corresponding precisely to that of a modern low-drag airfoil (Fig. 4).

Something that Cayley and numerous others, until this century, found difficult to understand was why drag is reduced if the object is endowed with a long and tapering tail. In the absence of a picture in which gradual deceleration of the fluid in the rear resulted in little or no separation and associated eddy formation, it appeared as though what matters in streamlining is not the front, but mainly the design of the rear, leading the biologist Vogel to quip that as Hamlet put it, "There's a divinity that shapes our ends, Rough-hew them how we will" (2).

I come now to lift, which is an extraordinary, almost magical, force produced perpendicular to the fluid flow when a shape like a bird's wing encounters it nearly edge on. It is far greater in magnitude than the associated drag, just as for the wheel, and is what holds the bird or airplane up against gravity when made equal to its weight. Its generation was properly understood only after the development of circulation theory by many clever minds in the early years of this century.

Lift is also the force of choice for efficiently producing thrust to counter the drag of a body moving in a fluid. The blades of a propeller and the screw of a ship are devices that work this way. In nature, all of the fastest continuously swimming animals in the open ocean move their crescent-shaped or so-called lunate tails, which have an airfoil cross section, in a precise pattern to produce high forward thrust. Penguins swim with their wings, also using a motion that produces lift-based thrust. They fly in the water exactly as birds fly in the air. To understand how in addition to providing lift for countering gravity the wing of a bird also produces thrust for propulsion

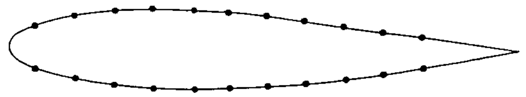

FIG. 4. A comparison of Cayley's sketch of the cross-section of a trout with a modern low-drag airfoil section. Dots indicate trout (adapted from figure 4 in ref. 1).

[†] This is a strange corruption of the Tamil word Kattumaram, which literally means, and refers to, a craft made of a few tapered logs tied together and widely used by fishermen on the coast of South India.

[‡] There are, interestingly, creatures other than humans who also sail. *Physalia*, a colonial coelenterate uses its crest as a sail, and the fishing spider *Dolomedes*, lifts its second pair of legs from the surface on windy days to be carried across the water. But most incredible is the *Velella*—another colonial coelenterate—which has an obliquely mounted airfoil-shaped pneumatophore with which it can sail to windward.

by using lift forces, I recommend the excellent book on this subject, *Bird Flight* (3) and references therein. Finally, the principles of lift generation by insects is full of surprises and a fascinating subject by itself.

Fluids and Friction

This is perhaps the moment to return to a point made earlier regarding the absolute need of friction to generate forces required for locomotion. It is just as true for fluids as for solids, that if you want to push against them you had better have friction somewhere. Feeling the force caused by motion of a fluid like air or water past our bodies is very much an every-day experience, and was surely the reason that d'Alembert was never taken seriously by practical experimenters in aviation. I refer to an apparent paradox advanced by the French mathematician more than 200 years ago, showing that there will be no net pressure on a solid past which a fluid flows (1). But the theorists could not brush it aside as casually, because it came in the way of the development of a consistent mathematical treatment of airfoil theory. It was Prandtl who finally settled this problem by his breakthrough in recognizing the existence and role of the boundary layer (1). This region, no matter how thin, is always there at any fluid–solid interface and allows viscosity to play its role in providing the required "grip."

An important second form of drag peculiar to real-world airfoils is that associated with producing lift and is the price paid for sustentation. This induced drag, as it is called, decreases rapidly with increase in speed for reasons closely analogous to the propulsion problem discussed earlier. The higher the forward speed, the more the mass (and the less the downward velocity) of the air deflected to provide the lift, and hence the less the energy required to deflect it. These two varieties of drag with inverse dependences on speed together lead to something peculiar to flight, namely two optimum speeds for a given input of energy, one which maximizes range and the other endurance in the air. It is at the former that both a jumbo jet and a migrating bird would fly when crossing oceans, and at the latter when an aircraft is in a holding pattern or a bird is searching for food over an area.

The above arguments in connection with induced drag should provide a hint as to why thrust based on lift (rather than on drag) involves a greater mass of fluid and has greater propulsive efficiency. To appreciate what a marvelous job nature has done with flying, one need only note that there are small migratory birds that fly thousands of miles without "refueling," an unthinkable feat for any land or water animal. A sparrow, which is identical in mass and metabolic rate to a mouse, flies an order of magnitude faster than a mouse runs, and so has a minimum cost of transport an order of magnitude lower than that of a mouse.

The wing is nature's wheel and can as freely convert potential to kinetic energy and vice versa. I used to think that nature had to resort to a mechanism so hard to understand physically, because it could not produce something with a true mechanical rotary joint, for almost obvious reasons. But I was wrong, and nature does make creatures with wheels, but not for rolling on hard ground. They live in a strange world, which we must now enter to know what they are and how they work.

A Sticky World

The creatures I refer to are bacteria and the special interest in them is because they live in a world dominated by viscosity, where inertia is negligible compared with drag that now varies directly with speed. A powerful dimensionless parameter involving size, speed, density, and viscosity whose value tells you what is likely to happen when solid and fluid move with respect to each other, is called Reynold's number. A duck flying at its usual speed will have a value of over 100,000, whereas the swimming bacterium will have the reciprocal of this number.

At such incredibly low Reynold's numbers life and locomotion are totally counterintuitive, as described picturesquely by Steven Vogel in his marvelous book (2) on the physical biology of flow, and from which I have borrowed generously for this paper. This is the world of flowing glass and creeping metals where flows, miraculously, become reversible. The effect of stirring three times clockwise can be undone by stirring three times counter-clockwise. Streamlining becomes a fine way to increase drag, circulation around an airfoil for lift is almost impossible, and turbulence is unimaginable.

The biologist Berg has compared the motion of a bacterium to that of a person trying to swim through asphalt (5). If it suddenly stopped rotating its flagellum, it would coast to a stop in a distance much less than the diameter of a hydrogen atom (2). The picture this evokes is one of incredible difficulty, requiring enormous strength to overcome. But it does not quite convey another aspect, namely the peculiar, technical, near-impossibility of generating thrust when viscosity reigns supreme.

In connection with the generation of lift and drag forces we saw the importance of the boundary layer wherein viscosity provided the required purchase. The trouble now is all purchase and no release, as when trying to shake off a particularly sticky piece of paper. Any of the usual forms of swimming that work so well for large creatures like humans would, at the end of any complete cycle of motions of the arms and legs, restore the swimmer to the same position as at the start of that cycle. You cannot kick back fluid that sticks to you like glue.

It is this aspect of the difficulty of locomotion for these diminutive creatures that was addressed by no less a giant in fluid mechanics than G. I. Taylor (4). He showed that the slantways motion of a long and thin cylinder like a flagellum would generate a force at an angle to its track because of the difference in drag for motions with the axis parallel and perpendicular to the flow. The rotation of a helical tail thus would provide thrust along the axis of the helix. Taylor worked out the full theory and even made a working model with twisted rubber bands where the tail went round and round but without rotation, like the pedal of a bicycle; because, and I quote Taylor, "rotation is a type of distortion which is impossible in a living organism" (4).

But evolution should never be underestimated, and Berg and Anderson showed that the flagella did rotate rigidly driven by a reversible rotary motor at their base (5). As this does not change the thrust produced, nature's choice of rigid rotation is presumably for other advantages. The conceptual difficulty of having true rotation was simply that of maintaining the physical connections to supply blood, nutrients, and nerve impulses to any extremities that can rotate without restraint with respect to the rest of the body. But in this strange microscopic world diffusion through membranes can provide the necessary nutrients and oxygen to keep the motor going.

On the matter of the energy required to move to the next meal, the famous physicist Purcell calculated that if the bacterium wished to increase by 10% the food supply that diffusion brings to it if stationary, it would have to move almost 25 times as fast as it usually does (2). Remembering that drag is proportional to speed, this would require about 600 times more power from the motor. As only Vogel could have put it, "We've encountered the equivalent of a casual cow who, after eating, just waits for the local grass to regrow" (2).

In Thick Air

We now have at least a rudimentary picture of locomotion in different media to venture attempting comparisons, but before we do that I would like to go back to vehicles on wheels and ask a question deferred earlier, namely what happens at high speeds. If the resistance to the motion of cars were caused only by rolling friction, the maximum speed would be roughly proportional to the power of the engine (when geared with the appropriate ratio). But as seen in the amazing plot of Fig. 5, the power required goes up as the cube of the speed in the range of the diagram—say above 120 km/h. This is dramatic evidence that above this speed rolling friction becomes negligible and the main resistance to be overcome is all air drag.

What is even more interesting is that the effective area causing drag is the same for all the vehicles, roughly equal to three-quarters of a square meter, and far less than their frontal area. This tells us how important streamlining is, and to what extent it has been implemented, given the constraint that if a car were shaped like a trout, it would either be far too long or have no useful volume to speak of. More importantly, it tells us that the lift/drag ratio collapses rapidly at higher speeds, when air drag rears its ugly head, and partly explains why the recent land-speed record-breaking car has an engine with 75 megawatts of power. This is more than what is required by the cube law, but at the supersonic speeds achieved fresh terms enter the drag equation.

An even more striking demonstration that the air is not as thin as it is transparent is the 200-m bicycle speed record over the years as shown in Fig. 6. Until 1974 all efforts went into improving the tires and the gears with only microscopic annual improvement. At this point they woke up to streamlining with immediate results (7). Remember the cube law and also that a racing cyclist is pushing aside one-half to 1 ton of air a minute, which is why members of a team benefit from riding one behind the other, just as birds do by flying in a V formation. It has been calculated that by flying in formation, 25 birds could get an increase of 70% in distance traveled for a given expenditure of energy, and it has been observed that the role of leader is changed over on a rota system during the flight.

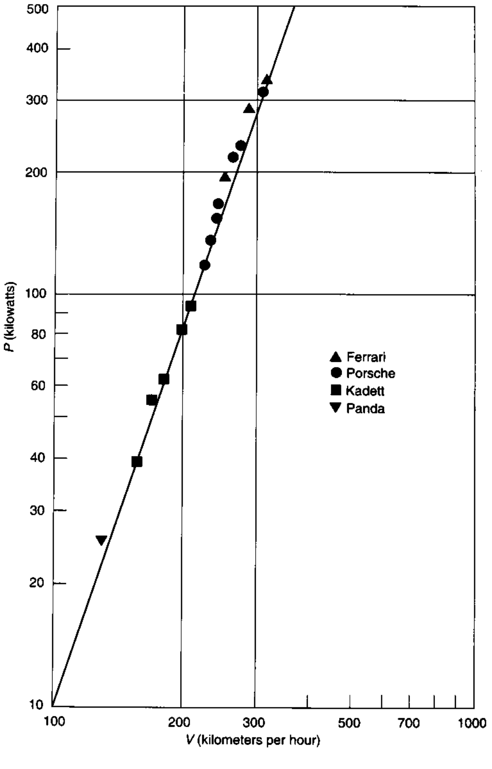

FIG. 5. Engine power plotted against maximum speed for four makes of automobiles. The diagonal is steep because power increases as the third power of the speed. To go twice as fast requires an 8-fold increase in engine power. [Reproduced with permission from ref. 6. (Copyright 1992 and 1996, Henk Tennekes. Reprinted by permission of MIT Press.)]

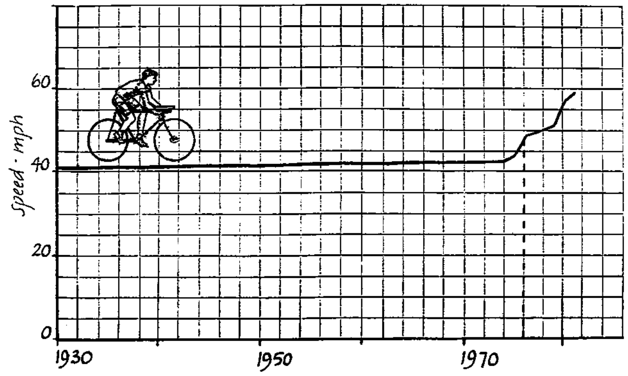

FIG. 6. Speed record over the years for a cyclist over a 200-m distance after a flying start: steady slow improvement, followed by the revolution of streamlining. [Reproduced with permission from ref. 7. (Copyright 1987, Philip and Phylis Morrison. Reprinted by permission of Random House, Inc.)]

To Speed or Not

Water, though occasionally transparent, is definitely thicker than air, and the force required to move water craft diminishes with decrease in speed, and at an even faster rate. A horse that would need a very smooth road and good wheels to pull a 3-ton wagon can tow a 30-ton barge at walking speed. If you were willing to go slow enough to reduce the Reynold's number to of order unity, the force required to tow anything depends only on the density and viscosity, and for water is 10^{-4} dynes. Ships taking their time to cross the oceans therefore must give the absolutely lowest cost per ton-mile, if fuel was all that was counted. But time is also money in more ways than one, not least when considering the capital cost investment. And speed has value from many different points of view, including the importance of being there physically and not just virtually.

This issue was confronted head-on in a famous article (8) written almost 50 years ago by Gabrielli (the director of the Fiat airplane factories in Italy) and von Karman. They began by saying that the history of technique and engineering testifies to the irresistible urge of humanity to go faster and faster and appropriately titled their article "What price speed?" They pointed out the difficulty of finding a measure of the comparative economy of locomotion because it is impossible to find a general measure for the value of speed in human life, and whose appreciation depends more on one's philosophy of life than on engineering science.

Having said all of this, they proceeded to lay the foundation of any such future analysis and to draw conclusions that qualitatively remain valid even after nearly half a century. As seen in their diagram (Fig. 7), ships are the most economical single vehicles at low speed. They also showed that for medium speeds the terrestrial, and for high speeds the aerial vehicles represent the optimum cases. Among numerous other perceptive remarks, they noted that convoys should be considered separately from single vehicles, especially trains that would have far less air drag per unit weight because of their high length to cross-section ratio. The speed of modern Japanese and French trains has far exceeded what was possible at the time of writing of their article, and I would like to illustrate their point with an example. We have seen in Fig. 5 that in the case of automobiles a 3-fold increase in speed from 100 to 300 km/h entailed a 27-fold increase in engine power. For the identical 3-fold increase in speed, the sleek French high-speed train (TGV) requires only a 3-fold increase in power, namely 12 megawatts as opposed to 4 megawatts for an old-fashioned train of similar length doing 100 km/h. It is for such vehicles whose rolling friction would dominate over air drag even at high speeds that magnetic levitation makes sense; a pilot

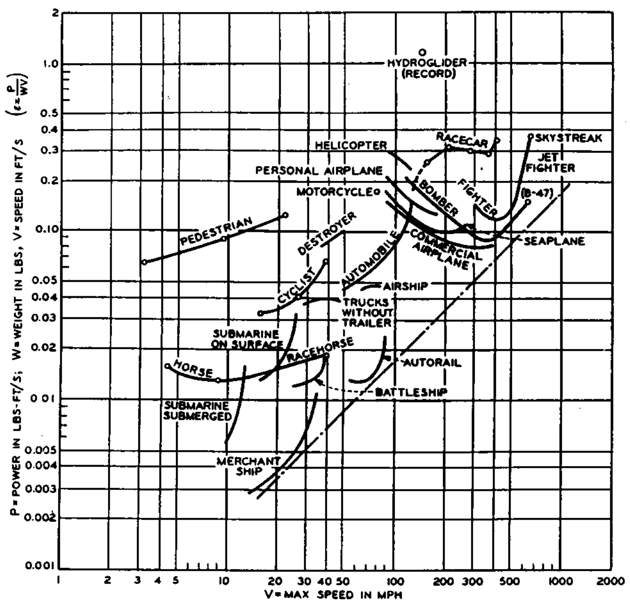

FIG. 7. The energy cost of transport in different types of single vehicles. [Reproduced from ref. 8 with permission. (Copyright 1950, American Society of Mechanical Engineers International.)]

MAGLEV train in Japan recently reached 400 km/h and has a target speed of 550 km/h.

In the final section of their remarkable analysis Gabrielli and von Karman investigated the limitations to speed imposed by structural considerations. By introducing an appropriate structural parameter involving the allowable stress and specific weight of the construction material they showed that it is understandable that every class of means of transportation approaches a special limit beyond which no practical design is possible. They concluded presciently that high-rate titanium alloys, in development at the time of their writing, might substantially change the results of their analysis (8).

A recent attempt at bringing things up to date can be found in the delightful book *The Simple Science of Flight* (6), which deals with very much more than flight of all kinds. The conclusion of Tennekes, the author, stated very simply is that 200 km/h is the rough dividing line below which wheels make energetic sense but above which wings hold sway. To quibble a bit, my guess is that "wheel-less" magnetically levitated trains may in the future come to occupy the middle region up to 500 km/h to advantage. But for transoceanic passenger travel and light freight, a jumbo jet flying at 1,000-plus km/h and at 10-plus km altitude has no economic competition as Tennekes very convincingly argued. Explaining the remarkable optimization, some of it fortuitous, of this particular aircraft, he called the Boeing 747 the commuter train of the global village and noted that "it was possible to design it only after titanium alloys much stronger than the best steel and aluminum alloys appeared on the market" (6).

Muscling Forward

Twenty-five years after "What price speed?" (8), and almost as long ago, appeared another famous paper entitled "The energetic cost of moving about" (9). It was written by Vance Tucker, a biologist at Duke University, who began by asking why people should encumber themselves with a heavy apparatus such as a bicycle, particularly while going uphill. The invariable answer is, of course, that it is easier, but Tucker investigated why it should be so by looking into the way the muscles involved in locomotion work and the amount of energy expended.

To one like me who has been involved only with mechanical vehicles, some biological notions take getting used to, such as that the cost of transport is infinite when the animal is standing still. This is because the metabolic rate remains finite even when the animal is not moving. Looked at this way, the efficiency becomes finite when the speed is greater than zero and the cost of transport reaches a minimum value at some speed at which the animal can cover distance on the level with the least energy expenditure. For example, a human of 70 kg achieves this at a fast walk—just over 6 km/h. At this speed, the metabolic rate is about 450 W, which when jogging briskly goes up even faster than the speed with a consequent drop in efficiency.

Tucker's study involved a vast range of animals, and in Fig. 8, we see his plot of the minimum costs of transport for a variety of swimmers, fliers, and runners, as well as some man-made devices and different forms of human locomotion. Tucker noted that because the range of masses on the abscissa covers 12 orders of magnitude from a fruit fly to a freight train, it is not surprising

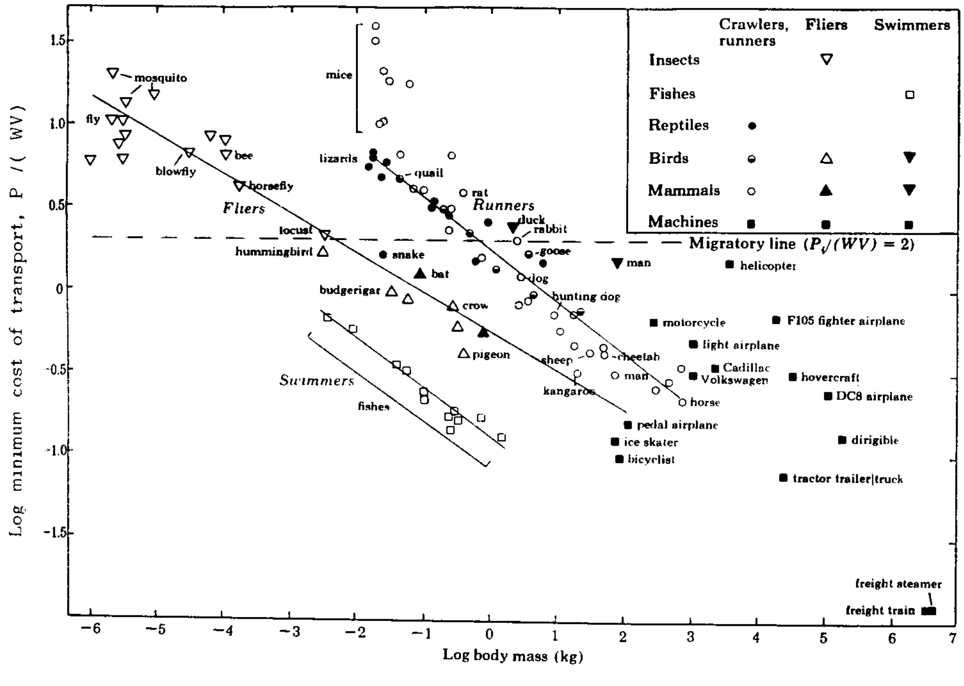

FIG. 8. The minimum energy costs of transport for a variety of swimmers, fliers, and runners as well as some man-made devices and different forms of human locomotion. [Reproduced from ref. 9 with permission. (Copyright 1975. Reprinted with permission from *American Scientist*.)]

that the minimum costs of transport vary widely. What is surprising, however, is that swimming, flying, and pedestrian animals fall roughly into separate groups irrespective of taxonomic status, as shown by the three lines in his plot. Excluding mice, a single line describes the minimum costs of transport for runners varying in size from small lizards and running birds to horses. Similarly, a single line fits the data for swimming fishes and another for fliers ranging in size from a fruit fly to the largest of muscle-powered fliers, a pedal airplane. Within each category, the more massive the animal the less its cost of transport. Among the three categories the line for swimming fishes represents the lowest cost, with fliers next and runners the most uneconomical. What can we learn from this amazing collection of data on animal and other locomotion?

In Three Dimensions

The first point to note is that most fish have internal mechanisms that adjust their density to become neutrally buoyant. Thus, expenditure of energy is not required to support them in the water, but only to push them through it. I already have remarked on the quality of nature's streamlining of fish, but have not mentioned that it is quantitatively so good that profile drag is a negligible addition to that caused by skin friction. The total is effectively equal to the resistance that a flat plate, of surface area equal to that of the fish, would feel when moving edgewise through the water at the same speed. As in the case of boats, the magnitude of this drag (proportional to the square of the speed) can be made arbitrarily small by decreasing the speed. But as pointed out above, the existence of a metabolic rate even for fish at rest must lead to an optimum speed that will depend both on the mass of the fish, and the dependence of metabolic rate on mass.

In a very famous paper written more than 60 years ago, Kleiber (10) showed that the basal, or resting, metabolic rate for animals over the vast range from mice to elephants scales closely as $M^{0.75}$. This amazing relationship also has been found to apply to aquatic species in more recent determinations (11). Small variations from this rule will make a negligible difference for my purpose here, and in Fig. 9, I have plotted both the static and dynamic contributions to the metabolic rate of swimming fish. Together they lead to a minimum cost of transport that scales approximately as $M^{-0.3}$, the slope of the line for fishes in Fig. 8.[§] And this minimum is achieved at an optimum speed that is practically independent of the mass. For the assumptions made here (see Fig. 9 legend) it scales as the 36th root of the mass, an interesting, but inconsequential, exponent.

The main point I wish to emphasize is that the low minimum cost of transport, and its scaling with body mass, is a natural consequence of physical laws and the low basal metabolic activity of cold-blooded fish. Proper warm-blooded swimmers are not represented in Fig. 8, but they have been discussed by Peters (11), who showed that they would fall on a line between those for fliers and runners in Tucker's plot. As remarked by Peters, the high basal metabolic rate for these so-called homeotherms makes motion virtually no more tiring for them than standing still, and "may contribute to the playfulness so apparent in captive whales, seals, and otters" (11).

Moving up in Fig. 8, the next line is for fliers. It excludes, naturally, soaring in thermals and gliding and such other forms of flight that derive energy from sources other than metabolic activity. One of Tucker's most interesting experiments was with a small parrot trained to fly freely in a wind tunnel while wearing a mask, so its power input could be measured during flight at various speeds (9). I already have mentioned that all flying machines should have two optimum speeds corresponding to maximum range and maximum endurance. Tucker's budgerigar demonstrated this beautifully as seen in Fig. 10. The balancing here is between the power required to be supported in the air and that to be pushed forward through it, at different speeds. As the resting metabolic rate for birds can be as low as a tenth of that during flight, it hardly plays a role in the establishment of the optimum speeds. Air is thinner than water, but birds and bats, like airplanes, have to fly fast enough to feel it sufficiently thick to support their weight, at which speeds the cost of transport becomes greater than in swimming. A great deal of work by many investigators has gone into understanding the aerodynamics and energetics of bird flight, and as Tucker, who has contributed immensely to this field, remarks, "The line for flying birds in the figure can be predicted" (9).

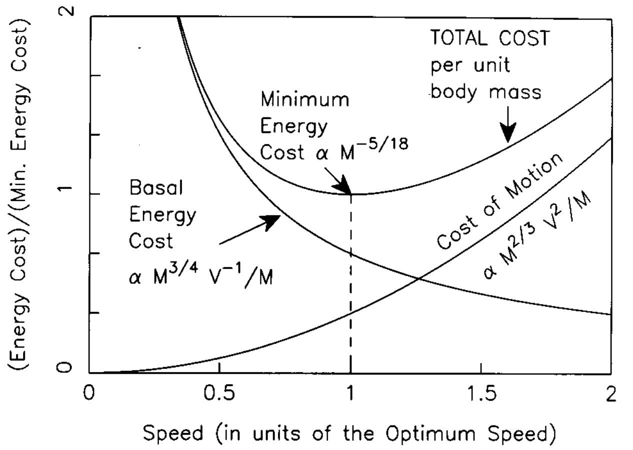

FIG. 9. The energy cost of transport over unit distance versus speed for swimming fishes. For transport of mass M over unit distance at uniform speed V, the basal cost is A/V and that for motion is BV^2, where A is assumed proportional to $M^{3/4}$ (metabolic rate), and B proportional to $M^{2/3}$ (surface area for self-similar shapes) and includes conversion efficiencies. Then, for unit body mass, the minimum energy cost is proportional to $M^{-5/18}$ (the slope in Tucker's plot) and is achieved at an optimum speed, which, for the above assumptions, is effectively independent of the mass, actually proportional to $M^{1/36}$. (It would be truly independent of the mass if the dependences of A and B on mass had a common exponent, say γ. The minimum energy cost then would be proportional to $M^{(\gamma-1)}$.)

Legging It

Pedestrian animals like ourselves are constrained to move on, or just above, an uneven two-dimensional surface that supports our weight whenever one or more of our limbs rests on the ground. It is true that we are immersed in the atmosphere, but still air contributes totally negligible resistance to movement at the normal speeds of most walkers and runners. Therefore, except for the initial acceleration, motion on the level and at constant speed should require energy only to overcome friction with the ground. Recalling the permanent overhead of a basal metabolic rate, and its high value for warm-blooded creatures, the cost of transport for land animals should decrease with increasing speed up to a point, and be the lowest instead of the highest as seen in Tucker's plot.

Note however from the same plot, that a human who is physically clumsy compared with most of nature's other creations, actually achieves the lowest cost of transport ever measured for an animal, by equipping himself with a bicycle. Also that one can do almost as well on ice skates. I already have remarked on the very low friction associated with wheels on level surfaces, and runners on ice, in connection with sail-powered craft. With these two devices one can propel oneself at speeds where lowering the head to reduce air resistance makes a difference, and when the cost of transport reaches the

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[§] Apart from the red aerobic muscles used for steady movement, most fish, like other vertebrates, also are equipped with anaerobic white muscles. But as these are used only for rapid acceleration or in bursts of speed they do not affect the discussion.

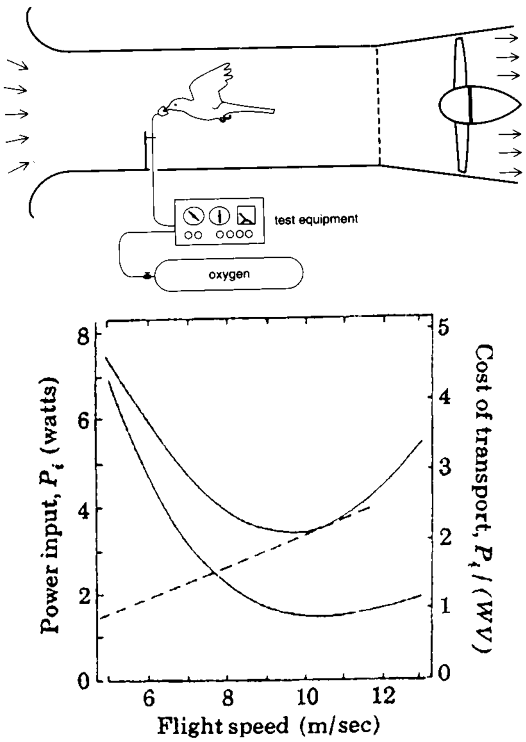

FIG. 10. (*Upper*) Vance Tucker's budgerigar in the wind tunnel. [Reproduced with permission from ref. 6. (Copyright 1992 and 1996, Henk Tennekes. Reprinted by permission of MIT Press.)] (*Lower*) The curves show the dependence on speed of power input (upper solid line), and energy cost of transport (bottom solid line), for the budgerigar in level flight. The energy cost of transport has its minimum at a higher speed than for the power input, and corresponds to the value where a dashed line drawn through the origin of the axes for power input and flight speed is tangent to the corresponding curve. [Reproduced from ref. 9 with permission. (Copyright 1975. Reprinted with permission from *American Scientist*.)]

ultimate low. Can this mean that the frictional loss between feet, paws or hooves and the ground is enormous? Not at all, because if one's foot does not slip when walking, its action approximates rolling, and very little energy is lost underfoot. What then is the meaning of the high cost of transport for walkers and runners because it is not likely that nature made a botch of them all.

There are two questions that need answers here. Where did the energy go, and what was it that was being optimized? I see them as related and shall attempt to answer them together. Unlike swimming or flying, which could be along a straight line connecting origin and destination, walking or running is the negotiation of the terrain in between with obstacles of a variety of shapes, and all possible sizes from pebbles to mountains. The limbs, and their extremities, of mountain goats jumping from crag to crag, or of monkeys swinging from branch to branch were undoubtedly optimized to increase maneuverability, and the ability to get there, with less regard to extra metabolic cost. Even more energy expensive than the steady-state aerobic values of Fig. 8 are the sprints of predators like cheetahs, and those of the antelopes that try to escape them, both highly occasional and that call on anaerobic muscles.

The near-zero (9) locomotory efficiency of warm-blooded walkers is simply caused by the fact that the energy expended in accelerating the body by contracting a muscle at the beginning of a step is lost by stretching it at the end of the step, and doing what biologists call "negative" work (because the displacement is opposite to the direction of the force). More recent work (12) has shown that some fraction of the energy can be stored elastically and recovered for the next step, but this does not affect the line of reasoning here. What was optimized as I see it, is the ability to stop dead or change direction (e.g., to avert possible danger), rather than letting momentum carry you along unwillingly like on a bicycle without brakes. For all the needs of life on land, efficiency in moving must be of less importance than having the incredible control and maneuverability of animals with legs, which vehicles on wheels or skates could hardly ever have.

An interesting historical development related by Gould (13) is that by the sixth century AD, 2,000 years after generals of the biblical armies rode on chariots, wheels as a means of transportation virtually disappeared from Moroco to Afghanistan and were replaced by camels. As pack animals, they were more efficient than carts pulled by draft animals (even by camels), could ford rivers, traverse rough ground, required far less manpower to tend them than if with wagons, and had great endurance and longevity. The committee supposedly charged with designing a horse did remarkably better than generally appreciated. Finally, as a space-age example of this kind of reversion, we have planetary exploration vehicles designed with legs instead of wheels to cope with very uneven terrain.

As Tucker explained (9), an alternative strategy for the efficient use of muscles is to prevent them from stretching at all. This can be accomplished by means of a mechanism that converts the downward velocity component of the body's center of mass at the end of one step cycle to the upward component at the start of a new cycle by applying a force to the center of mass at right angles to its direction of motion, doing no work in the process. This is the principle used by birds and bicyclists (and skaters) to attain high muscular efficiencies during locomotion. Tucker concluded by saying that the cost of transport on a bicycle is low because active muscles are not stretched while pedaling and mean muscle efficiency is about 0.25, nearly its maximum value. Thus do humans move along a level surface with the same muscular efficiencies that swimming and flying animals achieve naturally.

Locomotion at its best is thus the story of wheels and wings.

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