



# Cognitive resource allocation determines the organization of personal networks

Ignacio Tamarit<sup>a,b,1</sup>, José A. Cuesta<sup>a,b,c,d</sup>, Robin I. M. Dunbar<sup>e,f</sup>, and Angel Sánchez<sup>a,b,c,d</sup>

<sup>a</sup>Grupo Interdisciplinar de Sistemas Complejos, Departamento de Matemáticas, Universidad Carlos III de Madrid, 28911 Leganés, Madrid, Spain; <sup>b</sup>Unidad Mixta Interdisciplinar de Comportamiento y Unidad Social Universidad Carlos III de Madrid (UC3M)-Universidad de Valencia (UV)-Universidad de Zaragoza (UZ), Universidad Carlos III de Madrid, 28911 Leganés, Madrid, Spain; <sup>c</sup>Institute for Biocomputation and Physics of Complex Systems, University of Zaragoza, 50018 Zaragoza, Spain; <sup>d</sup>Institute UC3M-Banco Santander of Financial Big Data, Universidad Carlos III de Madrid, 28903 Getafe, Madrid, Spain; <sup>e</sup>Department of Experimental Psychology, University of Oxford, Oxford OX3 7LG, United Kingdom; and <sup>f</sup>Department of Computer Science, Aalto University, FI-00076 Aalto, Finland

Edited by H. Russell Bernard, University of Florida, Gainesville, FL, and approved April 24, 2018 (received for review November 3, 2017)

**The typical human personal social network contains about 150 relationships including kin, friends, and acquaintances, organized into a set of hierarchically inclusive layers of increasing size but decreasing emotional intensity. Data from a number of different sources reveal that these inclusive layers exhibit a constant scaling ratio of  $\sim 3$ . While the overall size of the networks has been connected to our cognitive capacity, no mechanism explaining why the networks present a layered structure with a consistent scaling has been proposed. Here we show that the existence of a heterogeneous cost to relationships (in terms of time or cognitive investment), together with a limitation in the total capacity an individual has to invest in them, can naturally explain the existence of layers and, when the cost function is linear, explain the scaling between them. We develop a one-parameter Bayesian model that fits the empirical data remarkably well. In addition, the model predicts the existence of a contrasting regime in the case of small communities, such that the layers have an inverted structure (increasing size with increasing emotional intensity). We test the model with five communities and provide clear evidence of the existence of the two predicted regimes. Our model explains, based on first principles, the emergence of structure in the organization of personal networks and allows us to predict a rare phenomenon whose existence we confirm empirically.**

comply with the (local) organization of the ego networks—just as any physical object must be consistent with its atomic composition. Our work therefore contributes to the literature and development of social physics—started in the 19th century by Comte and Quetelet—along the lines proposed in refs. 20 and 21 and other works aiming to study quantitative theories that yield testable predictions (see, e.g., ref. 22 for a detailed summary of the field before physicists themselves started to contribute to it). As we show below, our model not only accounts for the layered structure previously mentioned, but also predicts the existence of a contrasting regime. Indeed, we will see that depending on the relation between cost and available relationships, an inverted structure may arise in which layers with larger emotional content are also larger in size.

## Model Description

In a population of  $N$  individuals, relationships (links) can be established out of a set of  $r$  different categories (that we later refer to as “layers”) according to the strength of the links. This is the problem that we want to model, but in its bare bones, this is simply that of distributing a certain number of balls (links) in urns (layers)—in effect, a multinomial distribution. Of itself, this distribution yields no structure whatsoever but it is a reasonable

quantitative sociology | personal networks | complex systems

The analysis and modeling of social networks are a widely studied topic. A number of models have been proposed across different disciplines such as statistical physics and computer science (1), economics (2–4), statistics (5), or sociology (6, 7). All these models aim to explain commonly observed properties of social networks, including community structure, high clustering, degree correlations, etc. To this end, the focus is set on the macroscopic properties of the networks while keeping as simple as possible the assumptions made about the individuals involved. The macroscopic observables emerge then as a consequence of the interactions among the constituents of the system.

However, some of the most robust findings about human social networks go beyond these macroscopic quantities, being concerned with the size and organization of the individuals’ personal networks. These studies suggest that, among humans, an individual typically deals with about 150 relationships including kin and friends (8–12). These relationships are further organized into a set of hierarchically inclusive layers of increasing size with decreasing emotional intensity (8, 12–16) whose sizes follow a characteristic sequence with a scaling ratio close to 3 (15): 5, 15, 50, 150. Although the overall size of the networks has been connected to our cognitive capacity (17), no theoretical explanation has been given for the layered structure and the consistent scaling ratio even though there is considerable evidence for their existence from many different media sources (18, 19).

In what follows we develop a mathematical model of the building blocks of social systems (social atoms) that should serve to connect the individual and collective perspectives of human societies. Indeed, whatever the (global) social structure is, it must

## Significance

**The way we organize our social relationships is key to understanding the structure of our society. We propose a quantitative theory to tackle this issue, assuming that our capacity to maintain relationships is limited and that different types of relationships require different investments. The theory accounts for well-documented empirical evidence on personal networks, such that connections are typically arranged in layers of increasing size and decreasing emotional content. More interestingly, it predicts that when the number of available relationships is small, this structure is inverted, having more close relationships than acquaintances. We provide evidence of the existence of both regimes in real communities and analyze the consequences of these findings in our understanding of social groups.**

Author contributions: I.T., J.A.C., and A.S. designed research; I.T. and J.A.C. performed research; I.T., J.A.C., R.I.M.D., and A.S. analyzed data; I.T., J.A.C., R.I.M.D., and A.S. wrote the paper; and A.S. contributed datasets.

The authors declare no conflict of interest.

This article is a PNAS Direct Submission.

This open access article is distributed under [Creative Commons Attribution-NonCommercial-NoDerivatives License 4.0 \(CC BY-NC-ND\)](https://creativecommons.org/licenses/by-nc-nd/4.0/).

Data deposition: The code and the data used in this paper have been deposited at <https://github.com/Ignacio/Cognitive>.

<sup>1</sup>To whom correspondence should be addressed. Email: [ignacio.tamarit@uc3m.es](mailto:ignacio.tamarit@uc3m.es).

This article contains supporting information online at [www.pnas.org/lookup/suppl/doi:10.1073/pnas.1719233115/-DCSupplemental](http://www.pnas.org/lookup/suppl/doi:10.1073/pnas.1719233115/-DCSupplemental).

prior to assume as the default. In this setting, the probability that there are  $\ell_k$  balls in urn  $k \in 1, 2, \dots, r$  will be

$$P_0(\ell|N) = \frac{(N-1)!(r+1)^{-N+1}}{\ell_1!\ell_2!\dots\ell_r!(N-1-\ell_1-\ell_2-\dots-\ell_r)!}, \quad [1]$$

where  $\ell = (\ell_1, \ell_2, \dots, \ell_r)$ .

Let us now assume that there is a cost  $s_k$  associated to each ball placed in urn  $k$ . Urns are initially all alike, so without loss of generality we can sort them by decreasing costs,  $s_1 > s_2 > \dots > s_r$ . We now look for a probability distribution that is constrained to have a fixed average number of balls  $L$  and a fixed average amount of resources  $S$  to afford its costs; that is,

$$\sum_{k=1}^r \mathbb{E}(\ell_k) = L, \quad \sum_{k=1}^r s_k \mathbb{E}(\ell_k) = S. \quad [2]$$

To add this information to our prior, the procedure to follow is the maximum entropy principle (23, 24), as it is the only way to guarantee a posterior distribution that is compatible with the prior, compatible with the additional information, and unbiased (23, 25). The result is a distribution that measures the likelihood of different allocations of balls to urns with different costs

$$P(\ell|S, L, N) = \mathcal{B}(L, \mathcal{L}/N, N) \left( \frac{L}{\sum_k e^{-\mu s_k}} \right)^L e^{-\mu \sum_k s_k \ell_k}, \quad [3]$$

with  $\mathcal{B}(L, p, N) = \binom{N}{L} p^L (1-p)^{N-L}$  the binomial distribution and where  $\mu$ , the only parameter of the model, arises as the Lagrange multiplier associated to the constraint on the total resources in Eq. 2 (SI Appendix, Section 1C).

**Application to Ego Networks.** Note that the model presented is fully general and applies to any situation in which a certain number of items of any sort have to be assigned to some categories with different costs. However, its connection with the organization of links within ego networks is rather natural. Although relationships change over time [they strengthen or weaken, new ones are created, and some old ones fade (26)], each individual handles a certain average number of links  $L$  at any one time (13). These relationships are further organized into different layers (urns,  $\ell_k$ ) according to the emotional strength (or closeness) of the links (for example, refs. 18 and 27 and references therein). Additionally, studies of both offline and online

social networks indicate that time invested in interacting with individual alters seems to determine the emotional strength of the relationship (the higher the investment, the closer the relationship) (13, 28) and is thus largely responsible for their layered structure (8, 13, 14, 19, 27, 29, 30). These investments reflect the costs,  $s_k$ , that individuals have to make to create functional relationships. If we further assume a limited (cognitive) capacity  $S$  of individuals to handle relationships (8, 17), we have a problem to which the previous model applies. In what follows, we analyze which kind of testable predictions can be inferred from the organization of links in an ego network implied by Eq. 3.

**Results.** We explore the emergence of structure, calculating the expected number of links in each layer. The ratio of this quantity between consecutive layers is

$$\frac{\mathbb{E}(\ell_{k+1})}{\mathbb{E}(\ell_k)} = e^{\mu|\Delta s_k|}, \quad [4]$$

where  $|\Delta s_k| = |s_{k+1} - s_k| > 0$  is the cost difference between them.

Eq. 4 identifies two distinct regimes according to whether  $\mu > 0$  or  $\mu < 0$ :

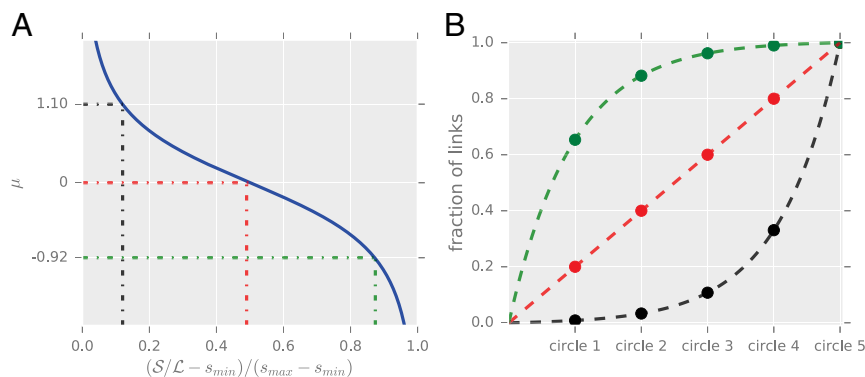
- If  $\mu > 0$ , then  $\mathbb{E}(\ell_{k+1}) > \mathbb{E}(\ell_k)$ , and the most expensive layers will be less populated than the less expensive ones. We call this the standard regime.
- If  $\mu < 0$ , then  $\mathbb{E}(\ell_{k+1}) < \mathbb{E}(\ell_k)$ , and the most expensive layer will be the most populated one. We call this the inverse regime.

Let us now consider that  $|\Delta s_k|$  is a constant so that costs  $s_k$  decrease linearly with  $k$ . We can then write  $s_k = s_1 - (s_1 - s_r)(k-1)/(r-1)$ , where  $r$  is the number of layers and  $s_1 > s_r > 0$ . In this scenario, the value of  $\mu$  is determined by the value of  $S/L$  according to (see SI Appendix, Section 1D, for details)

$$\frac{s_1 - S/L}{s_1 - s_r} = f(\hat{\mu}) \equiv e^{\hat{\mu}} \frac{(r-1)e^{r\hat{\mu}} - re^{(r-1)\hat{\mu}} + 1}{(r-1)(e^{r\hat{\mu}} - 1)(e^{\hat{\mu}} - 1)}, \quad [5]$$

where we define  $\hat{\mu} \equiv \mu(s_1 - s_r)/(r-1)$  for convenience. Note that the choice  $|\Delta s_k| = 1$  implies that  $\hat{\mu} = \mu$ , so we use both interchangeably.

Hence, which regime an individual belongs to depends on the ratio  $S/L$  (Fig. 1), and this in turn depends on the total number of social relationships that an individual has. If  $L$  is large, this structure will be standard. This is what has been



**Fig. 1.** The two regimes as a function of the mean cognitive cost allocatable per link. (A) Dependency of the parameter  $\mu$  with the ratio  $S/L$ . The blue line represents the typical dependency of the parameter with the mean cognitive cost  $S/L$  that an individual can spend in maintaining a link. As a reference, it has been computed with Eq. 5 for  $r = 5$  circles and  $\Delta s_k = 1$ , but it is representative of the expected behavior. Given a fixed cognitive capacity  $S$ , increasing  $L$  implies moving to the left in the graph. The particular value of  $S/L$  determines the value of  $\mu$ . Dotted lines represent example cases (fixed  $S$ ); in green, an individual with “few” alters (inverse regime;  $\mu = -0.92$ ); in red, the limit case (change of regime;  $\mu = 0$ ); and in black, an individual with “many” alters (standard regime;  $\mu = 1.10$ ). (B) Expected regimes as a function of  $\mu$ . The colors follow the specifications given in A. That is, the black dashed line represents the standard regime, the red one the limit case, and the green one the inverse regime. Solid circles represent the expected fraction of links in each circle for the different examples.





interaction seem to be restricted to a small number of individuals by social exclusion).

Note that all distributions (Fig. 2A and Fig. 3A–D) show a large dispersion, implying that the structure of circles is quite personal (and depends, among other things, on the individual's number of links). This confirms an earlier empirical finding suggesting that individuals allocate their social effort in quite different and consistent ways, such that each is characterized by a kind of “social fingerprint” (26). In fact, taking both results together, the parameter of our model may serve as a quantitative characterization of such a fingerprint.

Although we have presented these results in terms of layers or circles, a simple modification of the current model gets rid of the layers to classify ties in a continuum (*SI Appendix, section 1F*), thus reproducing what is typically seen in most personal social networks [i.e., individual alters can be listed in a continuous list of emotional closeness and/or contact frequency (9)]. The two structural regimes obtained when there are discrete layers also arise in this version. Thus, whether we view egocentric networks in terms of layers or as an ordered linear sequence of dyadic relationships simply reflects different (equally valid) ways of describing an individual's personal social network.

One possible criticism of these results is that they may be an artifact of the way the questions are posed in the surveys—people are usually asked to classify their relationships in predefined categories. However, a number of approaches have been taken and yield much the same pattern in different types of datasets: Online social networks such as those based on Twitter or Facebook as well as those based on phone calls (14, 19) yield exactly the same layered organization as we find in self-rated questionnaire-based ratings (8, 13, 15).

Another possible criticism could be that the data we used in our empirical validation were obtained using different methodologies (*Materials and Methods*). Nevertheless, there are examples in the literature that suggest that the influence of the different protocols is not significant. Studies with larger source populations (i.e., more choices available) (17, 37) have shown that, even when using an open-ended method, individuals list only about 10–30 people, and the structure found was still the standard regime. This is because imposing a cutoff on a standard network does not change it into an inverse structure: This is clear from Fig. 2B where a cutoff at, say, layer 2 or 3 would not change the form of the distribution into that shown in Fig. 2C (examples also given in ref. 26). Note that the name generator used with the groups of immigrants had a limit of 30 names (*Materials and Methods*), and we nonetheless found both standard and inverse regimes. Additionally, the data reported in the shipboard survey mentioned above (16), where individuals living in a boat were asked about their relationships with other members of the expedition (a protocol similar to that used with the community of students that we use in this paper), suggest average sizes of 14.6 and 26.7 individuals for the first and second circles, respectively, much as would be expected for an inverse regime. The inverse regime is precisely what we would have expected to emerge in this setting. Therefore, although further studies should investigate the impact of different protocols, there is no a priori reason to suppose that either methodology would bias the results in any particular direction.

It may be surprising that there are individuals whose ego networks show an organization opposite to what is typical in their contexts. There are several reasons why this might be so, all of which derive from the fact that the potential size of the ego network is constrained. As such, these are predictions of the model that could be tested. One is that an individual's cognitive capacity [the ability to manage many relationships, which is a function of an individual's brain size (17, 38, 39) or intellectual ability (40) or the time costs of investment in ties (28, 41)] is limited or because the available population is small (for geographical or, as in the case our immigrant samples, social reasons). Network size might also vary with personality differences. Introverts, for example, typically have significantly smaller egocentric social networks

than extroverts (36). In such cases, introverts have smaller but emotionally more intense relationships on average than extroverts or those with large networks, who seem to spread their available cognitive capital more thinly (9, 36). This seems to be due to a constraint on available social time that applies across all individuals (42).

More interestingly, perhaps, our model predicts how the increasing availability of online social networks may affect the way we handle our relationships. Since these technologies reduce the effective cost of maintaining some relationships, it should be easier for individuals to establish larger networks and this should promote the standard regime. However, if online relationships are cheaper to maintain because they obviate the costly business of physically meeting up with an alter (41), it follows that any increase in online network size will be associated with a reduction in average tie strength. This would incentivize weak relationships, which might well be another reason why the inverse regime has remained largely unnoticed until now.

Finally, from a socio-centric perspective, our model suggests a way to identify whether an interconnected set of individuals (i.e., a community in the technical network analysis sense) is “small” or not, namely according to the regime of their ego networks. Consider as a reference a layered structure  $\ell = (5, 10, 35, 100)$  (giving the typical structure of circles: 5, 15, 50, 150) and an arbitrary linear decrease in the costs. In such a setting we find that the change of regime happens at a network size of 88 and that there is a maximum network size of 220 [a value close to the maximum observed network size of  $\sim 250$  (8, 36)]. We also find that communities with sizes less than or equal to 55 members will have most of their contacts in the inner circle (thus, forming an absolutely cohesive group). This latter finding is of particular interest, because groupings of  $\sim 50$  occur frequently in small-scale traditional societies: This is the typical size of hunter-gatherer bands (overnight camp groups), a grouping of special functional importance in terms of foraging and protection against predators (43). It also represents the primary functional social grouping in personal social networks, being the set of alters to whom an ego devotes most of his or her social time and effort (9, 13). More interestingly, perhaps, communities built up on a mixture of the two regimes might exhibit quite different properties from the socio-centric point of view. They might also gel less well and hence be less stable. Exploring these differences may shed light, for instance, on our understanding of the internal structure of human societies and the reasons why natural communities fission when they do (44).

## Materials and Methods

**Reciprocity Survey.** In this survey (32), 84 students (60% female and 40% male) from a major Middle Eastern university volunteered to participate. Each participant was presented with a list of the other 83 participants and was asked about his/her relationship with each one of them. The question we are interested in was stated as follows: “How close are you to this person?”. And the options were the following: “0, I do not know this person”; “1, I recognize this person but we never talked”; “2, acquaintance (we talk or hang out sometimes)”; “3, friend”; “4, close friend”; and “5, one of my best friends.” For each participant we store the number of answers of each type in an array  $(\ell_k)$ , so that  $\ell_{6-k}$  is the number of type  $k$  answers. These numbers are our representation of the layers. For the analysis presented in this paper we excluded the cases scored with either 0 (no relation whatsoever) or 1. The latter are excluded for two reasons: (i) Recognizing someone but having never talked with him or her hardly counts as a meaningful relationship, and (ii) there surely are other people outside this sample that the surveyed subjects recognize but never talked to, but are not part of the survey (limited to the 84 students) (see *SI Appendix, Section 3, and Fig. S2*, for a complete set of figures considering five instead of four layers).

**Communities of Immigrants.** The data were collected in a similar way between November 2008 and April 2009 in all four immigrant studies, using the open source software EgoNet. In the case of the Bulgarians, the following name generator was used (33): “Tell us about 30 people who you know on a first name basis, with whom you have had contact in at least the last

two years and who you could contact again if necessary. It is important that all categories of contacts (family, friends, workmates [ . . . ]) be represented." For the remaining three communities, the name generator was (34, 35) the following: "Tell us 30 people you know by name, and vice versa. It can be everyone. Try to mention people important for you, but also other people not so close but whom you meet frequently. Try to use pseudonyms, but be sure you can recognize them later." In both cases each participant rated the perceived closeness of his/her relationship with each alter. The options were as follows: "1, not close at all"; "2, not very close"; "3, quite close"; "4, close"; and "5, very close." With this information we create an array as before.

**Code Implementation and Data Availability.** All numerical analysis is carried out in Python with the packages `scipy.optimize` and `scipy.integrate` (see the documentation for details). The code and the data used in this paper are available at <https://github.com/1gnaci0/Cognitive>. The original data from the Chinese, Sikh, and Filipino communities (34) are available at [visone.info/wiki/index.php/Signos\\_\(data\)](http://visone.info/wiki/index.php/Signos_(data)). The data from the community of students (32) can be found at [dx.doi.org/10.1371/journal.pone.0151588](https://doi.org/10.1371/journal.pone.0151588). The original data from the community of Bulgarians are not publicly available, but we provide an anonymized version with the information relevant to our work.

**Estimate of the Parameter.** The maximum-likelihood estimate for  $\mu$  is obtained by numerically solving the equation  $\frac{L_1}{L} = (r-1)f(\mu)$ , where  $L \equiv \sum_{k=0}^{r-1} \ell_{k+1}$ ,  $L_1 \equiv \sum_{k=0}^{r-1} k\ell_{k+1}$ , and  $f(\mu)$  is given by Eq. 5 (see *SI Appendix, Section 1E*, for a full description of the above expressions). We used `fsolve` with tolerance  $10^{-6}$  for the relative error between two consecutive iterates. **Limit cases.** The model presents singularities when all of the relationships happen to be in either the first layer [then  $f(\mu) = 0$ , which holds for  $\mu \rightarrow -\infty$ ] or the last layer [then  $f(\mu) = 1$ , which holds for  $\mu \rightarrow +\infty$ ]. The

data from the reciprocity survey include one individual (no. 80) with this sort of structure, so we excluded this datum from our analysis.

**Confidence interval.** To find the  $1 - \delta$  confidence interval we have to compute the cumulative distribution (see *SI Appendix, Section 1E*, for details)  $G(t|\ell) = \int_{-\infty}^t P(\mu|\ell) d\mu$ , which involves integrating

$$F_t(R) \equiv \int_0^t \left( \frac{1 - e^{-\mu}}{1 - e^{-\mu r}} \right)^L e^{-\mu R} d\mu = \frac{1}{R} \int_0^{tR} \left( \frac{1 - e^{-z/R}}{1 - e^{-zr/R}} \right)^L e^{-z} dz. \quad [7]$$

For finite values of  $t$  we use quad. For  $t \rightarrow \infty$  we evaluate the integral using a Gauss-Laguerre quadrature with 150 points. The extremes of the confidence interval  $[t_1, t_2]$  are obtained by solving  $G(t_1|\ell) = \delta$  and  $G(t_2|\ell) = 1 - \delta$ . To that end, we use `fsolve` with tolerance  $10^{-6}$ . The results presented in this paper consider  $\delta = 0.025$  (95% confidence interval).

**Numerical stability.** Overflows in Eq. 7 due to the exponentials are avoided by evaluating the logarithm of the integrand. The singularity at  $\mu = 0$  is avoided by Taylor expanding  $e^{-\mu r}$  and  $e^{-\mu}$  up to third order. The singularity at  $\mu = 0$  of Eq. 6 is avoided by using the Taylor expansion  $\chi_k \approx k/r + (k/2r)(e^\mu - 1)(k - r)$  for  $|e^\mu - 1| \leq 10^{-6}$ .

**ACKNOWLEDGMENTS.** We thank José Luis Molina for sharing and discussing with us the data of the communities of immigrants. I.T., J.A.C., and A.S. were supported in part by Fundación Banco Bilbao Vizcaya Argentaria through Grant Los Números de Dunbar y la Estructura de las Sociedades Digitales: Modelización y Simulación; by Ministerio de Economía, Innovación y Competitividad (Spain) through Grants FIS2015-64349-P VARIANCE (Ministerio de Economía y Empresa/Fondo Europeo de Desarrollo Regional, Unión Europea); and by the European Commission through FET Open Research and Innovation Action 662725 Bridging the Gap: From Individual Behaviour to the Socio-Technical Man and FET Proactive RIA 640772 Distributed Global Financial Systems for Society. R.I.M.D. was supported by the European Research Council Advanced Investigator through Grant 295663.

- Toivonen R, et al. (2009) A comparative study of social network models: Network evolution models and nodal attribute models. *Soc Netw* 31:240–254.
- Goyal S (2012) *Connections: An Introduction to the Economics of Networks* (Princeton Univ Press, Princeton).
- Vega-Redondo F (2007) *Complex Social Networks* (Cambridge Univ Press, Cambridge, UK), Vol 44.
- Jackson MO (2010) *Social and Economic Networks* (Princeton Univ Press, Princeton).
- Snijders TA (2011) Statistical models for social networks. *Annu Rev Sociol* 37:131–153.
- Wasserman S, Faust K (1994) *Social Network Analysis: Methods and Applications* (Cambridge Univ Press, Cambridge, UK), Vol 8.
- Carrington PJ, Scott J, Wasserman S (2005) *Models and Methods in Social Network Analysis* (Cambridge Univ Press, Cambridge, UK).
- Hill RA, Dunbar RI (2003) Social network size in humans. *Hum Nat* 14:53–72.
- Roberts SG, Dunbar RI, Pollet TV, Kuppens T (2009) Exploring variation in active network size: Constraints and ego characteristics. *Soc Netw* 31:138–146.
- Haerter JO, Jamtveit B, Mathiesen J (2012) Communication dynamics in finite capacity social networks. *Phys Rev Lett* 109:168701.
- Gonçalves B, Perra N, Vespignani A (2011) Modeling users' activity on Twitter networks: Validation of Dunbar's number. *PLoS One* 6:e22656.
- Dunbar RI (2014) The social brain psychological underpinnings and implications for the structure of organizations. *Curr Dir Psychol Sci* 23:109–114.
- Sutcliffe A, Dunbar R, Binder J, Arrow H (2012) Relationships and the social brain: Integrating psychological and evolutionary perspectives. *Br J Psychol* 103:149–168.
- Dunbar RI, Arnaboldi V, Conti M, Passarella A (2015) The structure of online social networks mirrors those in the offline world. *Soc Netw* 43:39–47.
- Zhou WX, Sornette D, Hill RA, Dunbar RI (2005) Discrete hierarchical organization of social group sizes. *Proc R Soc Lond B Biol Sci* 272:439–444.
- Bernard HR, Killworth PD (1973) On the social structure of an ocean-going research vessel and other important things. *Social Sci Res* 2:145–184.
- Powell J, Lewis PA, Roberts N, García-Fiñana M, Dunbar RI (2012) Orbital prefrontal cortex volume predicts social network size: An imaging study of individual differences in humans. *Proc R Soc Lond B Biol Sci* 279:2157–2162.
- Kawachi I, Berkman LF (2001) Social ties and mental health. *J Urban Health* 78:458–467.
- Mac Carron P, Kaski K, Dunbar R (2016) Calling Dunbar's numbers. *Soc Netw* 47:151–155.
- Bernard HR, Killworth PD (1979) Why are there no social physics. *J Steward Anthropological Soc* 11:33–58.
- Pentland A (2015) *Social Physics: How Social Networks Can Make us Smarter* (Penguin, New York).
- Lazarsfeld PF (1961) Notes on the history of quantification in sociology—trends, sources and problems. *Isis* 52:277–333.
- Jaynes ET (2003) *Probability Theory: The Logic of Science* (Cambridge Univ Press, Cambridge, UK).
- Jiménez-Martín M, Tamarit I, Rodríguez-Laguna J, Korutcheva E (2017) Null models for social hierarchical structure. arXiv:1701.07428v3.
- Caticha A, Giffin A (2006) Updating probabilities. *AIP Conference Proceedings* (American Institute of Physics, Melville, NY), Vol 872, pp 31–42.
- Saramäki J, et al. (2014) Persistence of social signatures in human communication. *Proc Natl Acad Sci USA* 111:942–947.
- Dunbar R (2018) The anatomy of friendship. *Trends Cogn Sci* 22:32–51.
- Oswald DL, Clark EM, Kelly CM (2004) Friendship maintenance: An analysis of individual and dyad behaviors. *J Social Clin Psychol* 23:413–441.
- Roberts SB, Dunbar RI (2015) Managing relationship decay. *Hum Nat* 26:426–450.
- Hall JA (2018) How many hours does it take to make a friend? *J Soc Pers Relat*, 10.1177/0265407518761225.
- Granovetter MS (1977) The strength of weak ties. *Social Networks*, ed Leinhardt S (Academic, New York), pp 347–367.
- Almaatouq A, Radaelli L, Pentland A, Shmueli E (2016) Are you your friends friend? Poor perception of friendship ties limits the ability to promote behavioral change. *PLoS One* 11:e0151588.
- Mestres SG, Molina JL, Hoeksma S, Lubbers M (2012) Bulgarian migrants in Spain: Social networks, patterns of transnationality, community dynamics and cultural change in Catalonia (Northeastern Spain) 1. *Southeast Europe* 36:208–236.
- Molina JL, Petermann S, Herz A (2015) Defining and measuring transnational social structures. *Field Meth* 27:223–243.
- Molina JL, Pelissier F (eds) (2010) *Les Xarxes Socials de Sikhs, Xinesos i Filipins a Barcelona* [The social networks of Sikh, Chinese, and Filipinos in Barcelona] (Fundació ACSAR, Barcelona). Catalan.
- Pollet TV, Roberts SG, Dunbar RI (2011) Extraverts have larger social network layers. *J Individual Differ* 32:161–169.
- Dunbar RI, Spoor M (1995) Social networks, support cliques, and kinship. *Hum Nat* 6:273–290.
- Kanai R, Bahrami B, Roylance R, Rees G (2012) Online social network size is reflected in human brain structure. *Proc R Soc B* 279:1327–1334.
- Kwak S, Joo W-t, Youm Y, Chey J (2018) Social brain volume is associated with in-degree social network size among older adults. *Proc R Soc B* 285:20172708.
- Dunbar R (2015) Social networks and their implications for community living for people with a learning disability. *Int J Dev Disabil* 61:101–106.
- Pollet TV, Roberts SG, Dunbar RI (2013) Going that extra mile: Individuals travel further to maintain face-to-face contact with highly related kin than with less related kin. *PLoS One* 8:e53929.
- Miritello G, et al. (2013) Time as a limited resource: Communication strategy in mobile phone networks. *Soc Netw* 35:89–95.
- Lehmann J, Lee P, Dunbar R (2014) Unravelling the evolutionary function of communities. *Lucy to Language: The Benchmark Papers* (Oxford Univ Press, Oxford), pp 245–276.
- Dunbar RI, Sosis R (2018) Optimising human community sizes. *Evol Hum Behav* 39:106–111.