Supporting Information

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SI Materials and Methods

Extraction Enhancement, Λ_T. The decay rate of an ensemble of molecules with number density N_0 into the photonic resonance can be written as:

$$\Gamma^{\mathrm{PhC}}(\omega) = N_0 \int_{\mathrm{gain}} \Gamma(\mathbf{r},\omega)d\mathbf{r} = N_0 \frac{\pi\omega|\boldsymbol{\mu}|^2}{3\hbar\epsilon_0} \sum_{\mathbf{k},\omega_{\mathbf{k}}} \int_{\mathrm{gain}} |\mathbf{E}_{\mathbf{k},\omega_{\mathbf{k}}}(\mathbf{r})|^2 d\mathbf{r}\, \frac{1}{\pi} \frac{\Delta\omega_{\mathbf{k}}}{(\omega-\omega_{\mathbf{k}})^2+\Delta\omega_{\mathbf{k}}^2}. \quad \mathbf{[S1]}$$

This result is achieved by decomposing the Green's function of the system in the basis of normalized Bloch modes $\mathbf{E}_{\mathbf{k},\omega_{\mathbf{k}}}(\mathbf{r})$ and $(\mathbf{E}^{\mathrm{F}}(\mathbf{r}))$ with a finite lifetime characterized by $Q^{\mathrm{F}} = \frac{\omega_{\mathbf{k}}}{2\Delta\omega_{\mathbf{k}}}$, instead of true eigenmodes with an infinitely long lifetime (1). The following assumptions were adopted in Eq. **S1**: (*i*) The gain medium is uniform and isotropic, and (*ii*) the quality factors of the resonances are large enough so that different resonances at the same $\mathbf{k}$ are far apart from each other compared with their line width. Note that Eq. **S1** is essentially related to results in ref. 2. For a macroscopic PhC slab with an area of A as discussed in this paper, we can substitute $\sum_{\mathbf{k}}$ in Eq. **5** with $\frac{A}{(2\pi)^2}\iint \mathrm{d}k_x \mathrm{d}k_y$, as is commonly done in solid-state physics. When evaluating Eq. **5** at the resonance of the PhC, $(\omega = \omega_{\mathbf{k}})$, we get the differential on-resonance decay rate:

$$\Gamma^{\mathrm{PhC}}(\mathbf{k},\omega_{\mathbf{k}}) = \frac{AN_0|\boldsymbol{\mu}|^2}{6\pi^2\hbar\epsilon_0}\alpha^{\mathrm{F}}Q^{\mathrm{F}} = \frac{N_0\pi\omega_{\mathbf{k}}|\boldsymbol{\mu}|^2}{3\hbar\epsilon_0}\alpha^{\mathrm{F}} \times \mathcal{S}(\mathbf{k},\omega_{\mathbf{k}}) \quad \mathbf{[S2]}$$

as in Eq. **2**. Here, $\alpha^{\mathrm{F}} = \int_{\mathrm{gain}} |\mathbf{E}^{\mathrm{F}}(\mathbf{r})|^2 d\mathbf{r}$ is the energy confinement of the fluorescence resonant mode in the gain medium region. In Eq. **S2**, we clearly show the linear relation between the decay rate of molecules into crystal momentum $\mathbf{k}$ resonant frequency $\omega_{\mathbf{k}}$ and the corresponding SDOS. Note that only $N_{\mathrm{eff}} = N_0 A d_{\mathrm{eff}}^{\mathrm{F}}$ number of molecules can couple to the fluorescence resonance mode, although the molecule layer is assumed to be infinitely thick on top of the PhC. Compare this result with the situation where all these molecules are randomly oriented in the liquid with a refractive index of n, and radiating incoherently and uniformly into all directions, the extraction enhancement can be calculated with the results shown in Eq. **3**. Note that (*i*) the portion $\left(\frac{Q^{\mathrm{F}}}{Q_{\mathrm{r}}^{\mathrm{F}}}\right)$ of generated photons that can be radiated coherently to the far field is taken into account and (*ii*) the comparison is made for radiating directions close to the normal direction of the PhC. This expression is consistent with results reported by Boroditsky (3). Note that $d_{\mathrm{eff}}^{\mathrm{F}}$ is defined to be the thickness of the fluorescence mode in the gain medium, in which the region $\left(1-\frac{1}{e^2}\right)$ of total energy in the gain medium is stored. Here, $d_{\mathrm{eff}}^{\mathrm{F}}$ was determined to be 100 nm through FDTD calculations.

1. Novotny L, Hecht B (2006) *Principles of Nano-Optics* (Cambridge Univ Press, Cambridge, U.K.).
2. Messiah A (1976) *Quantum Mechanics* (Wiley, New York) Vol II, XXI.
3. Boroditsky M, et al. (1999) Spontaneous emission extraction and Purcell enhancement from thin-film 2-D photonic crystals. *Journal of Lightwave Technology* 17(11):2096–2112.