Supplementary Information for

Shaping the Branched Flow of Light through Disordered Media

Andre Brandstötter, Adrian Girschik, Philipp Ambichl and Stefan Rotter

Stefan Rotter
E-mail: stefan.rotter@tuwien.ac.at

This PDF file includes:

Supplementary text

www.pnas.org/cgi/doi/10.1073/pnas.1905217116

Supporting Information Text

Spatial basis. In order to cut out those parts of the transmission matrix t that describe the transmission from the entire left-hand lead to a specific region at the output lead, we use the transmission matrix t in a spatial basis. One element t_{ab} of this matrix then describes the transmission from point b at the input to point a at the output. A transformation between the transmission matrix in the modal basis (now called $t^{(m)}$) and the transmission matrix in the spatial basis (now called $t^{(s)}$) can be derived as follows: (i) We calculate the elements of the transverse position operator y (the y-axis is the transverse direction in our scattering geometry) in the modal basis

$$y_{ab} = \int_0^W dy\, \chi_a(y) \cdot y \cdot \chi_b(y), \qquad [1]$$

with $\chi_a(y)$ denoting the transverse mode profiles $\chi_a(y) = \sqrt{2/W} \cdot \sin(k_{y,a} y)$. (ii) We calculate the eigenbasis of the y-operator in Eq. (1). (iii) Using these eigenstates, we obtain the transmission matrix in coordinate space by $t^{(s)} = Y^\dagger t^{(m)} Y$, where Y contains the eigenvectors of the y-operator column-wise. In this basis, one row of $t^{(s)}$ describes the transmission from the entire incoming lead to only one peak on the right-hand side of the scattering geometry. Thus, if we are interested in the transmission matrix from the left lead to a designated region on the right side, we just need to cut out the corresponding rows falling outside the desired region.

Correlated disorder. The disorder potential is generated as follows: (i) For each grid point $\vec{r}$ of the scattering region a random number for the quantity $n^2(\vec{r}) - 1 =: x(\vec{r})$ between 0 and 1 is drawn uniformly. (ii) These random numbers are smoothed with a Gaussian correlation function

$$\begin{aligned} C(|\vec{r} - \vec{r}\,'|) &= \langle (x(\vec{r}) - \langle x(\vec{r})\rangle) \cdot (x(\vec{r}\,') - \langle x(\vec{r}\,')\rangle \\ &\propto \exp\left(\frac{-|\vec{r} - \vec{r}\,'|}{2\xi}\right) \end{aligned} \qquad [2]$$

with a certain correlation length ξ. (iii) The mean value is subtracted from all $x(\vec{r})$. (iv) We rescale all values by

$$\tilde{x}(\vec{r}) = x(\vec{r}) \cdot \left(\frac{a}{\sqrt{12\langle x^2(\vec{r})\rangle}}\right) + b \qquad [3]$$

such that $\langle \tilde{x}(\vec{r})\rangle = b$ and $\langle \tilde{x}^2(\vec{r})\rangle = a^2/12 + b^2$. For our calculation we choose $a = 0.21$ and $b = 0.105$. (v) In a last step we subtract the minimum value, i.e., $\tilde{x}'(\vec{r}) = \tilde{x}(\vec{r}) - \min(\tilde{x}(\vec{r}))$, to assure $\tilde{x}'(\vec{r}) > 0$ (since $n^2(\vec{r})$ has to be larger than 1) and, finally, obtain the refractive index by $n(\vec{r}) = \sqrt{1 + \tilde{x}'(\vec{r})}$. We end up with a correlated refractive index $n(\vec{r})$ characterized by a correlation length $\xi = 6 = 3\lambda$, minimum/maximum values: $\min(n(\vec{r})) = 1$ and $\max(n(\vec{r})) \approx 1.19$, a mean value $\mathrm{mean}(n(\vec{r})) \approx 1.1$ and a standard deviation $\mathrm{std}(n(\vec{r})) \approx 0.03$.