Supporting Information

William Nordhaus, Economic aspects of global warming in a post-Copenhagen environment

Downloadable version. Note that the model is available in an Excel version at the author's webpage at http://www.econ.yale.edu/~nordhaus/homepage/RICEmodels.htm.

Major Equations of the RICE 2010 Model. The following lists the major equations in the RICE-2010 model. We omit unimportant equations such as initial conditions. Note that subscript *k* represents regions.

(A.1) Welfare:

$$W = \sum_{k=1}^{12} \sum_{t=1}^{T\,max} \left(\varphi_{kt} \, U[c^k(t), L^k(t)] R(t) \right)$$

(A.2) Discount factor:

$$R(t) \;= (1+\rho)^{-t}$$

(A.3) Utility function:

$$U\,[c^k(t), L^k(t)] = L^k(t)[c^k(t)^{1-\alpha} \;/\; (1-\alpha)]$$

(A.4) Output before damages and abatement:

$$Y^k(t) = A^k(t)\, K^k(t)^{\gamma}\, L^k(t)^{1-\gamma}$$

(A.5) Abatement cost as fraction of output:

$$\Lambda^k(t) = Y^k(t)\, \theta_1(t) \mu^k(t)^{\theta_2}$$

(A.6) Climate damages as fraction of output

$$\Omega^k(t) = g^k[T(t), SLR(t), M_{AT}(t)] / \left(1 + g^k[T(t), SLR(t), M_{AT}(t)]\right)$$

(A.7) Output after damages and abatement

$$Q^k(t) = \Omega^k(t)[1 - \Lambda^k(t)] Y^k(t)$$

(A.8) Composition of output

$$Q^k(t) = C^k(t) + I^k(t)$$

(A.9) Per capita consumption

$$c^k(t) = C^k(t) \;/\; L^k(t)$$

(A.10) Law of motion of capital stock

$$K^k(t) = I^k(t) - \delta_K K^k(t-1)$$

(A.11) Industrial emissions

$$E_{Ind}{}^k(t) \;= \sigma^k(t)[1 - \mu^k(t)] Y^k(t)$$

(A.12) Carbon fuel limitations

$$CCum \geq \sum_{t=1}^{T\,max} \left[\sum_{k=1}^{12} E^k{}_{Ind}(t) \right]$$

(A.13) Total carbon emissions

$$E^k(t) = E^k{}_{Ind}(t) + E^k{}_{Land}(t)$$

(A.14) Dynamics of atmospheric carbon concentrations

$$M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1)$$

(A.15) Dynamics of carbon concentrations in biosphere and upper oceans

$$M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1)$$

(A.16) Dynamics of carbon concentrations lower oceans

$$M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1)$$

(A.17) Radiative forcings

$$F(t) = \eta\{log_2[M_{AT}(t) / M_{AT}(1750)]\} + F_{EX}(t)$$

(A.18) Global mean surface temperature

$$T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}$$

(A.19) Temperature lower oceans

$$T_{LO}(t) = T_{LO}(t-1) + \xi_4\{T_{AT}(t-1) - T_{LO}(t-1)]\}$$

(A.20) Sea level rise (thermal expansion, glaciers, ice sheets)

$$SLR(t) = SLR(t-1) + \left[\sum_{j=1}^{5} \pi_{1,j} + \pi_{2,j} T_{AT}(t-1) + \pi_{2,j}[T_{AT}(t-1) - \overline{T}^j_{AT}]\right]$$

Variable definitions and units. Endogenous variables are marked with asterisks. These omit the regional subscripts or superscripts for brevity.

$A(t)$ = total factor productivity (productivity units)

* $c(t)$ = per capita consumption of goods and services (2005 U.S. dollars per person)

* $C(t)$ = consumption of goods and services (trillions of 2005 U.S. dollars)

* $D(t)$ = damages from climate change (trillions of 2005 U.S. dollars)

$E_{Land}(t)$ = emissions of carbon from land use (billions of metric tons C per period)

* $E_{Ind}(t)$ = industrial carbon emissions (billion metric tons C per period)

* $E(t)$ = total carbon emissions (billion metric tons C per period)

* $F(t), F_{EX}(t)$ = total and exogenous radiative forcing (watts per square meter from 1900)

*$g^k[T(t), SLR(t), M_{AT}(t)]$ = damage function

* $I(t)$ = investment (trillions of 2005 U.S. dollars)

* $K(t)$ = capital stock (trillions of 2005 U.S. dollars)

$L(t)$ = population and proportional to labor inputs (millions)

* $\Lambda(t)$ = abatement cost as fraction of output

* $M_{AT}(t), M_{UP}(t), M_{LO}(t)$ = mass of carbon in reservoir for atmosphere, upper oceans, and lower oceans (billions of metric tons C, beginning of period)

* $\mu(t)$ = emissions-control rate (fraction of uncontrolled emissions)

$\sigma(t)$ = ratio of uncontrolled industrial emissions to output (metric tons C per output in 2005 prices)

* $\Omega(t)$ = damage function (climate damages as fraction of regional output)

* $\Lambda(t)$ = abatement cost function (abatement costs as fraction of regional output)

* $Q(t)$ = output of goods and services, net of abatement and damages (trillions of 2005 U.S. international dollars)

$SLR(t)$ = sea level rise (relative to 1990), meters

t = time (decades from 2001-2010, 2011-2020, …)

* $T_{AT}(t), T_{LO}(t)$ = global mean surface temperature, temperature upper oceans, temperature lower oceans (°C from 1900)

* $U\ [c(t), L(t)]$ = instantaneous utility function (utility per period)

* W = objective function in present value of utility (utility units)

* $Y(t)$ = output of goods and services, gross of abatement and damages (trillions of 2005 U.S. dollars)

Parameters

α = elasticity of marginal utility of consumption (pure number)

$CCum$ = maximum consumption of fossil fuels (billions metric tons carbon)

γ = elasticity of output with respect to capita (pure number)

δ_K = rate of depreciation of capital (per period)

η = temperature-forcing parameter (°C per watts per meter squared)

$\phi_{11}, \phi_{21}, \phi_{22}, \phi_{32}, \phi_{12}, \phi_{33}, \phi_{23}$ = parameters of the carbon cycle (flows per period)

$\xi_1, \xi_2, \xi_3, \xi_4$ = parameters of climate equations (flows per period)

ψ_1, ψ_2 = parameters of damage function

ρ = pure rate of social time preference (per year)

$R(t)$ = social time preference discount factor (per time period)

$Tmax$ = length of estimate period for model (60 periods = 600 years for most variables)

$\overline{T}_{AT}^{j}(t)$ = threshold temperatures for ice sheets in SLR equation (°C)

$\theta_1(t), \theta_2$ = parameters of the abatement cost function

φ_{kt} = Negishi parameters of the social welfare function

Time steps. The current model runs on 10-year time-steps. Variables are generally defined as flow per year, but some variables are in flow per decade. The transition parameters are generally defined as per decade. Users should check the Excel program to determine the exact definition of the time-steps. The basic model runs for 60 10-year periods. The SLR module projects atmospheric concentrations, temperature, and SLR to 3000.

Further Tables on RICE-2010 model Results. The following tables provide more detailed information on the results of the model runs.

Growth of net national income	2005-2055	2055-2105	2105-2205
	[Percent per year, logarithmic]		
US	2.04	1.08	0.29
EU	1.84	0.88	0.28
Japan	1.02	0.71	0.30
Russia	1.73	0.85	0.36
Eurasia	2.65	1.44	0.40
China	3.90	1.25	0.30
India	4.29	1.87	0.37
Middle East	3.59	1.69	0.27
Africa	4.99	2.33	0.30
Latin America	3.16	1.47	0.33
Other high income	1.99	0.79	0.27
Other developing Asia	4.10	2.10	0.37
World	**2.79**	**1.45**	**0.32**

Table ST1. Growth of Net National Income by region, baseline run

Net national income equals consumption plus the growth in the net capital stock.

Capping region:	Date of participation	Base year	Commitment year	Fraction of base year in commitment year	Further reductions tied to
US	2015	2005	2015	0.84	House bill
EU	2005	1990	1995	0.80	US
Japan	2005	1990	1995	0.94	US
Russia	2005	1990	2005	1.00	US
Eurasia	2020	1990	2020	1.00	US
China	2030	2030	2030	1.00	US
India	2040	2040	2040	1.00	US
Middle East	2050	2050	2050	1.00	US
Africa	2070	2070	2070	1.00	US
Latin America	2030	2030	2030	1.00	US
OHI	2015	2015	2015	1.00	US
Other non-OECD Asia	2040	2040	2040	1.00	US

Table ST2. Participation rates in Copenhagen Accord

	2000	2100	2200	2300
CO_2 concentrations (ppm)				
Base	369.5	748.0	1,250.0	1,227.6
Optimal	369.5	591.7	493.2	455.6
Limit T < 2 °C	369.5	453.7	417.4	398.4
Copenhagen: Full Trade	369.5	532.9	506.4	474.2
Copenhagen: No trade	369.5	530.8	483.2	463.6
Copenhagen: Rich only	369.5	676.4	808.9	726.6
Radiative forcings (W/m^2)				
Base	1.60	5.99	8.50	8.41
Optimal	1.60	4.42	3.41	2.97
Limit T < 2 °C	1.60	2.83	2.49	2.23
Copenhagen: Full Trade	1.60	3.77	3.55	3.19
Copenhagen: No trade	1.60	3.74	3.29	3.06
Copenhagen: Rich only	1.60	5.38	6.12	5.48
Temperature (°C from 1900)				
Base	0.83	3.51	5.72	6.56
Optimal	0.83	2.77	2.71	2.41
Limit T < 2 °C	0.83	2.00	1.92	1.82
Copenhagen: Full Trade	0.83	2.49	2.64	2.58
Copenhagen: No trade	0.83	2.48	2.51	2.43
Copenhagen: Rich only	0.83	3.20	4.52	4.37

Table ST3. Results for climate variables for different runs

Carbon prices	2015	2025	2035	2045	2055	2105
	(2005 prices per ton C)					
Optimal	37.96	65.50	89.50	117.76	155.55	408.48
Limit T $\leq$ 2 °C	79.04	142.25	226.00	348.97	521.78	903.69
Copen: Full trade	0.39	5.79	64.12	201.26	358.37	593.10
Copen: Rich only	0.39	12.40	41.76	64.14	64.11	27.68
	(2010 prices per ton C)					
Optimal	42.68	73.65	100.64	132.42	174.91	459.30
Limit T < 2 °C	88.87	159.95	254.12	392.40	586.70	1,016.13
Copen: Full trade	0.44	6.51	72.10	226.31	402.96	666.90
Copen: Rich only	0.44	13.94	46.96	72.12	72.09	31.13
	(2005 prices per ton CO2)					
Optimal	10.35	17.87	24.41	32.12	42.43	111.42
Limit T < 2 °C	21.56	38.80	61.65	95.19	142.33	246.50
Copen: Full trade	0.11	1.58	17.49	54.90	97.76	161.78
Copen: Rich only	0.11	3.38	11.39	17.50	17.49	7.55
	(2010 prices per ton CO2)					
Optimal	11.64	20.09	27.45	36.12	47.71	125.29
Limit T < 2 °C	24.24	43.63	69.32	107.04	160.04	277.18
Copen: Full trade	0.12	1.78	19.67	61.73	109.92	181.91
Copen: Rich only	0.12	3.80	12.81	19.67	19.66	8.49

Table ST4. Globally averaged carbon prices, alternative policy scenarios
[Prices are weighted by industrial CO2 emissions.]

Carbon price (2005 $ per ton carbon)	2015	2025	2035	2045	2055	2105
US	34.55	82.64	229.22	397.39	523.12	592.74
EU	0.00	74.04	196.79	476.89	706.78	878.72
Japan	184.87	198.12	352.71	532.04	708.96	836.46
Russia	0.00	0.00	0.00	39.13	141.26	289.42
Eurasia	0.00	0.00	0.00	11.79	114.17	297.16
China	0.00	0.00	69.54	201.85	317.52	417.29
India	0.00	0.00	0.00	213.96	467.48	711.35
Middle East	0.00	0.00	0.00	0.00	168.95	553.48
Africa	0.00	0.00	0.00	0.00	0.00	417.91
Latin America	0.00	0.00	210.00	528.51	757.51	897.81
OHI	0.00	20.82	160.61	361.20	527.40	648.55
Other	0.00	0.00	0.00	269.48	566.65	821.90
Global average	**10.13**	**22.28**	**80.29**	**209.41**	**354.51**	**570.71**

Table ST5. Carbon price for Copenhagen Accord with no trading

Emissions control rates (% of baseline)	2005	2015	2025	2035	2045	2055	2065	2075	2085	2095	2105
US	0.0	15.5	21.6	26.3	31.5	37.6	44.3	50.9	57.1	69.0	72.7
EU	0.0	12.2	16.9	20.6	24.6	29.4	34.7	39.9	44.7	53.9	56.9
Japan	0.0	12.2	16.9	20.6	24.6	29.4	34.7	39.9	44.7	53.9	56.9
Russia	0.1	19.5	27.0	33.0	39.4	47.1	55.6	63.8	71.5	86.4	91.1
Eurasia	0.1	19.5	27.0	33.0	39.4	47.1	55.6	63.8	71.5	86.4	91.1
China	0.0	17.9	24.8	30.3	36.2	43.3	51.0	58.6	65.6	79.3	83.6
India	0.0	13.9	19.3	23.6	28.1	33.7	39.7	45.6	51.1	61.7	65.1
Middle East	0.0	14.7	20.4	24.8	29.7	35.5	41.8	48.0	53.8	65.0	68.6
Africa	0.0	13.9	19.3	23.6	28.1	33.7	39.7	45.6	51.1	61.7	65.1
Latin America	0.0	12.7	17.6	21.5	25.6	30.7	36.2	41.5	46.5	56.2	59.3
OHI	0.0	13.9	19.3	23.6	28.1	33.7	39.7	45.6	51.1	61.7	65.1
Other developing Asi	0.0	13.2	18.4	22.4	26.8	32.1	37.8	43.4	48.6	58.8	62.0
Global	0.0	15.4	21.3	25.9	30.7	36.6	42.9	49.1	54.8	65.9	69.3

Table ST6. Emissions control rate, optimal case

Emissions control rates (% of baseline)	2005	2015	2025	2035	2045	2055	2065	2075	2085	2095	2105
US	0.0	2.1	9.7	28.7	49.8	64.3	73.3	77.1	82.5	86.6	89.5
EU	0.0	1.6	7.6	22.5	38.9	50.3	57.3	60.3	64.6	67.7	70.0
Japan	0.0	1.6	7.6	22.5	38.9	50.3	57.3	60.3	64.6	67.7	70.0
Russia	0.0	2.6	12.2	36.0	62.3	80.5	91.8	96.5	103.4	108.5	112.1
Eurasia	0.0	0.0	12.2	36.0	62.3	80.5	91.8	96.5	103.4	108.5	112.1
China	0.0	0.0	0.0	33.0	57.2	73.9	84.2	88.6	94.9	99.6	102.9
India	0.0	0.0	0.0	0.0	44.5	57.5	65.5	68.9	73.8	77.5	80.0
Middle East	0.0	0.0	0.0	0.0	0.0	60.6	69.1	72.7	77.8	81.7	84.4
Africa	0.0	0.0	0.0	0.0	0.0	0.0	0.0	68.9	73.8	77.5	80.0
Latin America	0.0	0.0	0.0	23.4	40.6	52.4	59.7	62.8	67.3	70.6	72.9
OHI	0.0	1.8	8.7	25.7	44.5	57.5	65.5	68.9	73.8	77.5	80.0
Other developing Asia	0.0	0.0	0.0	0.0	42.4	54.8	62.4	65.7	70.3	73.8	76.3
Global	0.0	0.8	3.4	17.7	35.7	53.0	56.2	69.8	73.4	75.7	77.9

Table ST7. Emissions control rate, Copenhagen Accord with full trading

Carbon price ($ per t C)	2005	2015	2025	2035
US	0.00	4.28	6.07	8.17
EU	0.00	5.55	7.75	10.40
Japan	0.00	1.69	2.26	2.81
Russia	0.00	0.10	0.10	0.10
Eurasia	0.00	0.53	0.75	0.98
China	0.00	6.81	9.87	13.77
India	0.00	5.05	7.42	10.55
Middle East	0.00	2.94	4.21	5.77
Africa	0.00	4.17	7.00	10.90
Latin America	0.00	2.85	4.28	6.00
OHI	0.00	2.71	3.63	4.72
Other	0.00	2.44	4.22	6.88
Global (emissions weighted)	0.00	4.17	6.02	8.31

Table ST8. Calculated carbon prices in Nash equilibrium

[Prices are US 2005 $ per ton carbon]

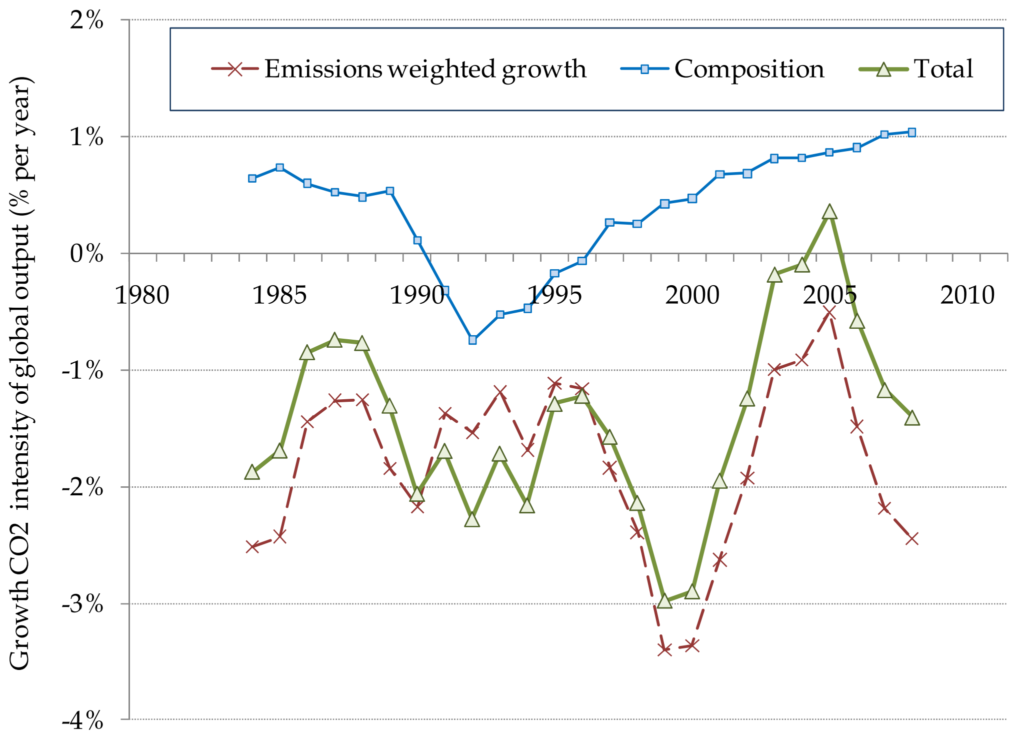

Figure SF1. Global rates of decarbonization
Emissions weighted growth is regional growth of emissions per unit output weighted by region's share of emissions. Composition is effect of changes in regional shares.

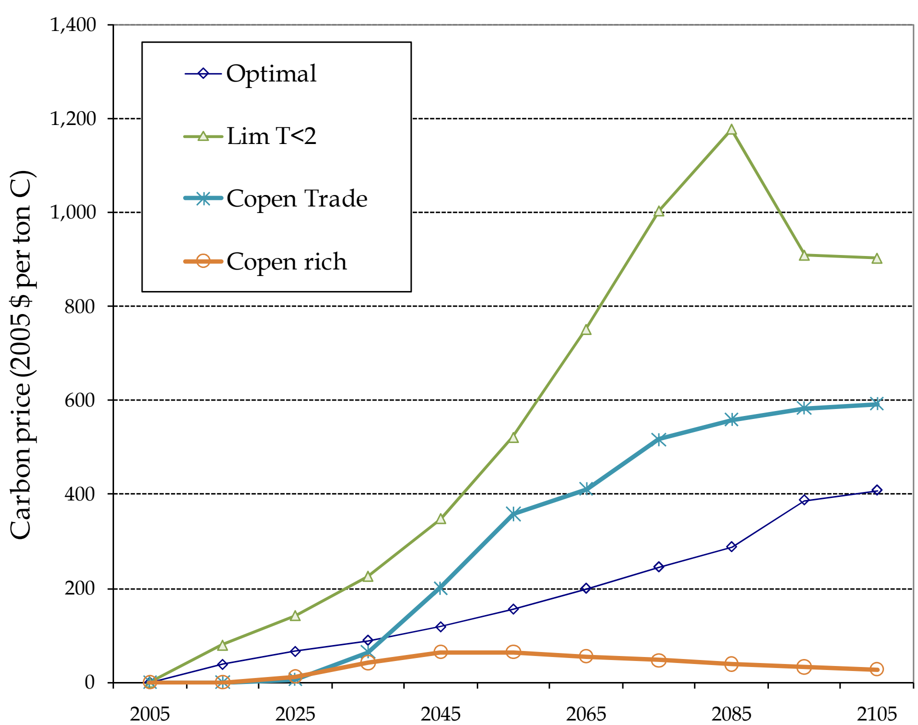

Figure SF2. Globally averaged carbon prices in different policy runs

Comparison with Earlier RICE/DICE model results. The following graphs provide comparisons with earlier versions of the RICE/DICE models. The first comprehensive model was published in 1994. Intermediate versions were in most case published, but in some cases were circulated or used for other published studies. All economic figures have been translated into 2005 U.S. dollars using the U.S. GDP price index.

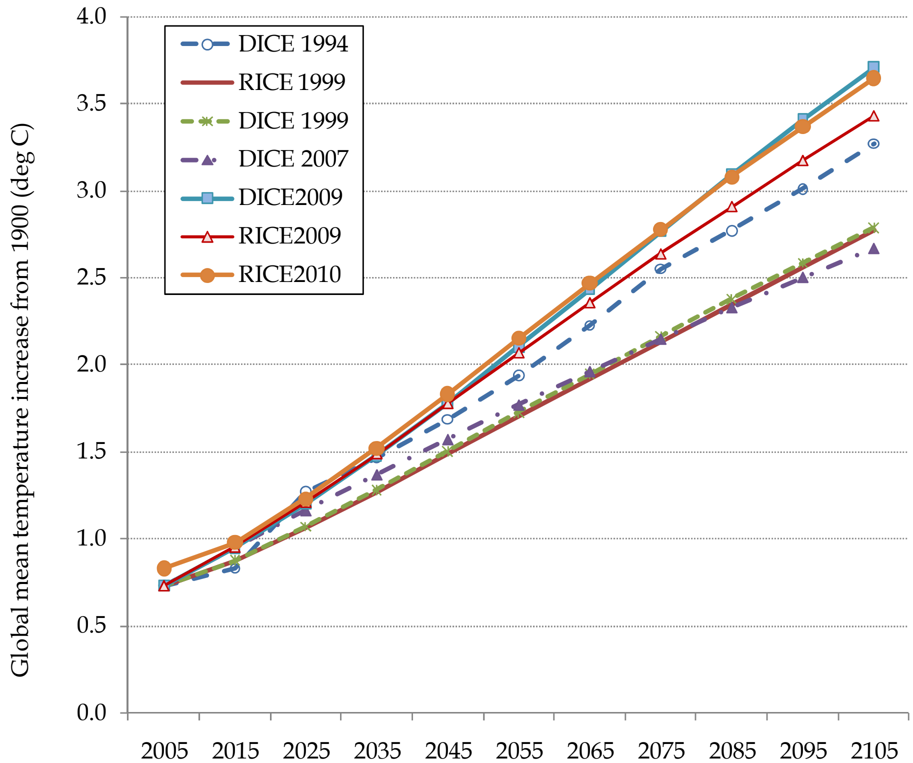

Figure SF3. Comparison of temperature projections, baseline runs, alternative vintages of RICE/DICE models

This shows the temperature projections from the first DICE model in 1994 through RICE-2010.

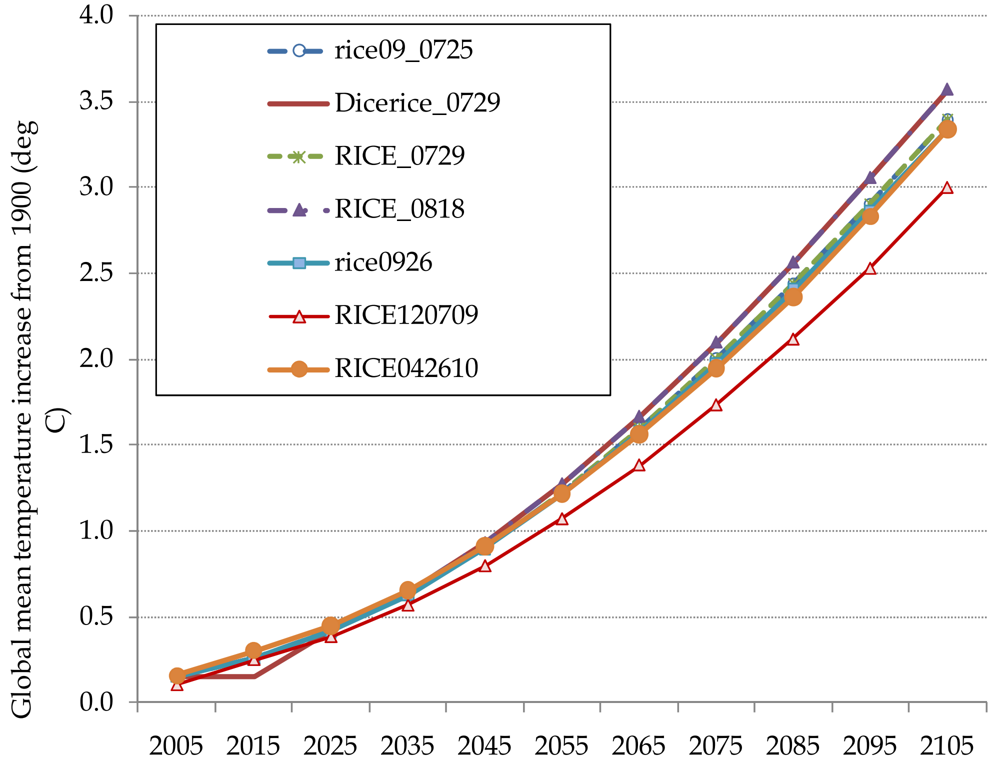

Figure SF4. Damages as percent of output, baseline, different vintages DICE/RICE models

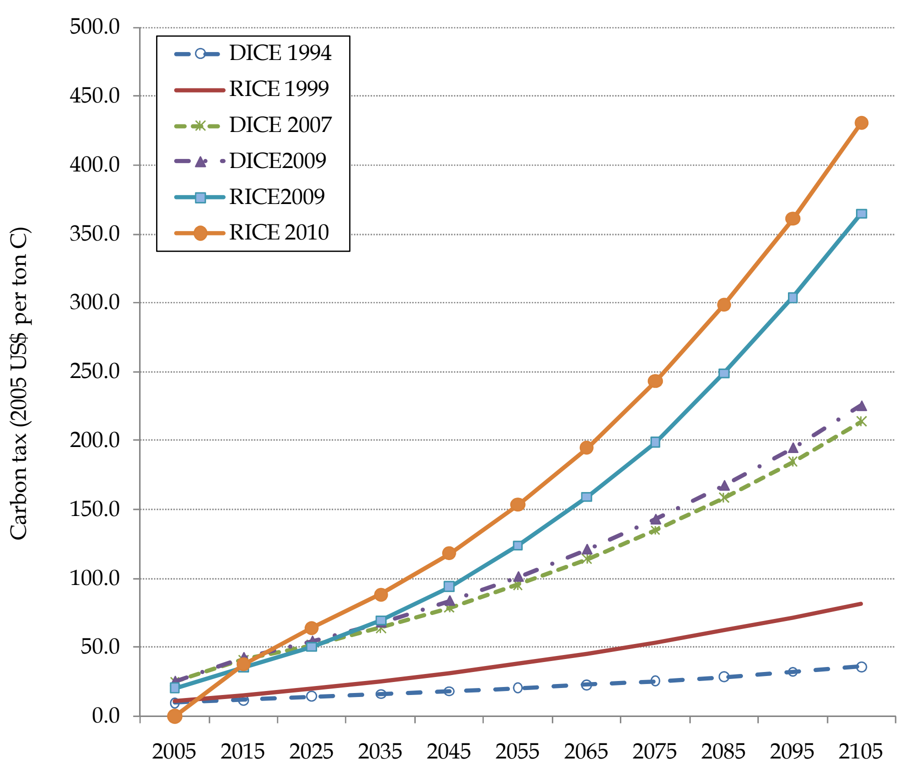

Figure SF5. Estimates of optimal carbon price, different vintages of DICE/RICE models, 1994-2010

Comparisons with EMF-22. Energy Modeling Forum 22 provided a model comparison of several integrated assessment models. The RICE 2010 model was not completed at that time and did not participate. The most useful comparison is the EMF "Reference" scenario, which is the modeler's projection with no climate change policies. A discussion of the EMF results is contained in reference (14) of the main article. Results for EMF-22 were made available by the organizers as an Excel file.

The results were available for 10 models: ETSAP-TIAM, FUND, GTEM, MERGE Optimistic, MERGE Pessimistic, MESSAGE, MiniCAM – BASE, POLES, SGM, and WITCH. Figure SF6 shows the RICE-2010 base projection along with the 10th, 50th, and 90th percentile of the EMF-22 runs.

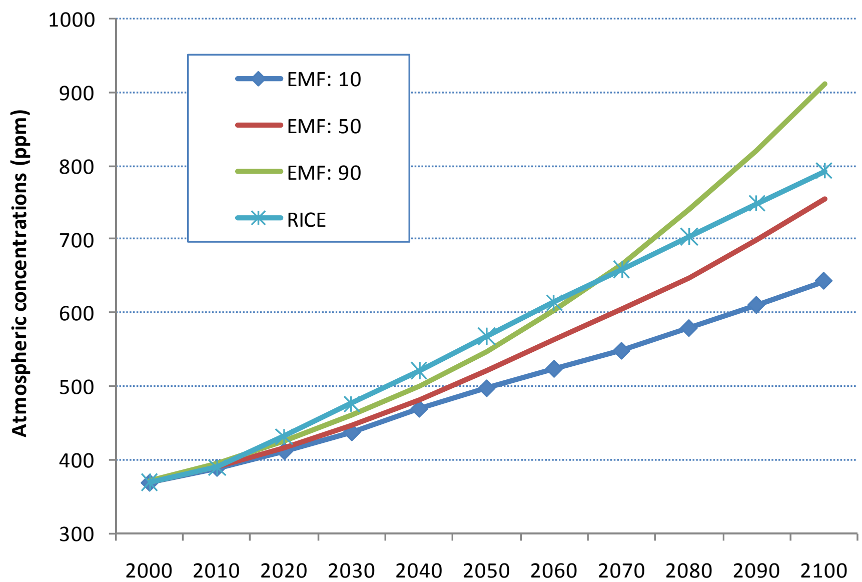

Fig SF6. Comparison between RICE-2010 and EMF-22 CO2 concentrations

Nash Equilibrium Calculations. The Nash equilibrium is calculated iteratively. The procedure starts with the base run. It then performs a full dynamic optimization of the carbon price region-by-region taking other regions' carbon prices as given. This technique is standard (17, 31 of the main article.) Estimates from earlier modeling results generally used the emissions-control rate rather than the carbon tax as the control variable, but this raises no issues of principle except convergence.

The step-by-step algorithm cycles through the regions until convergence is attained. Stability (change $< 0.5\%$) was attained in the third iteration. The carbon prices for Russia were constrained to be positive. Note that this procedure will overestimate the Nash equilibrium prices for multi-country regions (e.g., Africa) because it assumes they act as a coalition. On the other hand, if several regions act together (say, the high-income countries) as a coalition, the equilibrium prices would be higher.

Table ST8 shows the equilibrium prices for the first four periods with the first period constrained to be 0.

Carbon price ($ per t C)	2005	2015	2025	2035
US	0.00	4.28	6.07	8.17
EU	0.00	5.55	7.75	10.40
Japan	0.00	1.69	2.26	2.81
Russia	0.00	0.10	0.10	0.10
Eurasia	0.00	0.53	0.75	0.98
China	0.00	6.81	9.87	13.77
India	0.00	5.05	7.42	10.55
Middle East	0.00	2.94	4.21	5.77
Africa	0.00	4.17	7.00	10.90
Latin America	0.00	2.85	4.28	6.00
OHI	0.00	2.71	3.63	4.72
Other	0.00	2.44	4.22	6.88
Global (emissions weighted)	0.00	4.17	6.02	8.31

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