Supporting Information

Gao et al. 10.1073/pnas.1005720107

SI Text

Teleportation-Based controlled-NOT (C-NOT) Gate. Here, we describe in detail the scheme of a teleportation-based C-NOT gate. We give a step-by-step analysis of its implementation with our setup, shown in Fig. 2 in the main article.

We align each β-barium borate crystal carefully to produce a pair of polarization entangled photons i and j in the state

$$|\Psi^{+}\rangle_{ij}=\frac{1}{\sqrt{2}}(|H\rangle_i|H\rangle_j+|V\rangle_i|V\rangle_j). \quad \textbf{[S1]}$$

We use the method described in refs. 1–4 to prepare the cluster state $|\chi\rangle$. Initially, photons 3, 4, 5, and 6 are in the state

$$|\Psi^{+}\rangle_{34}\otimes|\Psi^{+}\rangle_{56}=\frac{1}{2}(|H\rangle_3|H\rangle_4|H\rangle_5|H\rangle_6+|H\rangle_3|H\rangle_4|V\rangle_5|V\rangle_6 +|V\rangle_3|V\rangle_4|H\rangle_5|H\rangle_6+|V\rangle_3|V\rangle_4|V\rangle_5|V\rangle_6). \quad \textbf{[S2]}$$

We direct photons 4 and 6 to the two input modes of a partially polarizing beam splitters (PPBSs), respectively. The transmission $T_H(T_V)$ of horizontally (vertically) polarized light at the PPBS is $1(1/3)$, and we thus get

$$\rightarrow\frac{1}{2}(|H\rangle_3|H\rangle_{4'}|H\rangle_5|H\rangle_{6'}+\frac{1}{\sqrt{3}}|H\rangle_3|H\rangle_{4'}|V\rangle_5|V\rangle_{6'} +\frac{1}{\sqrt{3}}|V\rangle_3|V\rangle_{4'}|H\rangle_5|H\rangle_{6'}-\frac{1}{3}|V\rangle_3|V\rangle_{4'}|V\rangle_5|V\rangle_{6'}). \quad \textbf{[S3]}$$

Here we have neglected terms with more than one photon in a single output mode of the PPBS, because in the experiment we postselect only terms that lead to a sixfold coincidence.

In order to symmetrize the state we place a PPBS ($T_H=1/3$, $T_V=1$) in each output mode of the PPBS and receive

$$\rightarrow\frac{1}{6}(|H\rangle_3|H\rangle_{4''}|H\rangle_5|H\rangle_{6''}+|H\rangle_3|H\rangle_{4''}|V\rangle_5|V\rangle_{6''} +|V\rangle_3|V\rangle_{4''}|H\rangle_5|H\rangle_{6''}-|V\rangle_3|V\rangle_{4''}|V\rangle_5|V\rangle_{6''}). \quad \textbf{[S4]}$$

This is already the desired four-qubit cluster state up to local unitary operations. To bring it to the desired form, we place half-wave plates—with an angle of 22.5° between the fast and the horizontal axis—into arms 3 and 4. This yields

$$\begin{aligned}&\rightarrow(|H\rangle_3|H\rangle_{4''}+|V\rangle_3|V\rangle_{4''})|H\rangle_5|H\rangle_{6''}+(|H\rangle_3|V\rangle_{4''}\\&+|V\rangle_3|H\rangle_{4''})|V\rangle_5|V\rangle_{6''}\\&=|\chi\rangle_{34''56''},\end{aligned} \quad \textbf{[S5]}$$

where we have neglected the overall prefactor 1/6 and we arrive at the desired ancillary four-photon cluster state $|\chi\rangle$ described in ref. 5.

Photons 1 and 2 constitute the input to our C-NOT gate. We assume that they are in a most general input state $|\Psi_{\mathrm{in}}\rangle_{12}$, where

$$|\Psi_{\mathrm{in}}\rangle_{ij}=\alpha|H\rangle_i|H\rangle_j+\beta|H\rangle_i|V\rangle_j+\gamma|V\rangle_i|H\rangle_j+\delta|V\rangle_i|V\rangle_j. \quad \textbf{[S6]}$$

The prefactors α, β, γ, and δ are four arbitrary complex numbers satisfying $|\alpha|^2+|\beta|^2+|\gamma|^2+|\delta|^2=1$. Before we proceed, let us define the desired output state after a C-NOT operation:

$$\begin{aligned}|\Psi_{\mathrm{out}}\rangle_{ij}=U^{\mathrm{C\text{-}NOT}}|\Psi_{\mathrm{in}}\rangle_{ij}&=\alpha|H\rangle_i|H\rangle_j+\beta|V\rangle_i|V\rangle_j\\&+\gamma|V\rangle_i|H\rangle_j+\delta|H\rangle_i|V\rangle_j.\end{aligned} \quad \textbf{[S7]}$$

The target qubit i is flipped on the condition that the control qubit j is in the state $|V\rangle$.

We can now express the combined state of all six photons in terms of Bell states for photons 1&3 and 2&5 and in terms of the desired output state $|\Psi_{\mathrm{out}}\rangle_{46}$ for photons 4&6 with corresponding Pauli operations:

$$\begin{aligned}&|\Psi_{\mathrm{in}}\rangle_{12}\otimes|\chi\rangle_{3456}=\\
&|\Phi^{+}\rangle_{13}|\Phi^{+}\rangle_{25}\ |\Psi_{\mathrm{out}}\rangle_{46} &&+|\Phi^{+}\rangle_{13}|\Phi^{-}\rangle_{25}\ \hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46}\\
&+|\Phi^{+}\rangle_{13}|\Psi^{+}\rangle_{25}\ \hat{\sigma}_x^4\hat{\sigma}_x^6|\Psi_{\mathrm{out}}\rangle_{46} &&+|\Phi^{+}\rangle_{13}|\Psi^{-}\rangle_{25}\ \hat{\sigma}_x^4\hat{\sigma}_x^6\hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46}\\
&+|\Phi^{-}\rangle_{13}|\Phi^{+}\rangle_{25}\ \hat{\sigma}_z^4\hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46} &&+|\Phi^{-}\rangle_{13}|\Phi^{-}\rangle_{25}\ \hat{\sigma}_z^4|\Psi_{\mathrm{out}}\rangle_{46}\\
&+|\Phi^{-}\rangle_{13}|\Psi^{+}\rangle_{25}\ \hat{\sigma}_x^4\hat{\sigma}_z^4\hat{\sigma}_x^6\hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46} &&+|\Phi^{-}\rangle_{13}|\Psi^{-}\rangle_{25}\ \hat{\sigma}_x^4\hat{\sigma}_z^4\hat{\sigma}_x^6|\Psi_{\mathrm{out}}\rangle_{46}\\
&+|\Psi^{+}\rangle_{13}|\Phi^{+}\rangle_{25}\ \hat{\sigma}_x^4|\Psi_{\mathrm{out}}\rangle_{46} &&+|\Psi^{+}\rangle_{13}|\Phi^{-}\rangle_{25}\ \hat{\sigma}_x^4\hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46}\\
&+|\Psi^{+}\rangle_{13}|\Psi^{+}\rangle_{25}\ \hat{\sigma}_x^6|\Psi_{\mathrm{out}}\rangle_{46} &&+|\Psi^{+}\rangle_{13}|\Psi^{-}\rangle_{25}\ \hat{\sigma}_x^6\hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46}\\
&+|\Psi^{-}\rangle_{13}|\Phi^{+}\rangle_{25}\ \hat{\sigma}_x^4\hat{\sigma}_z^4\hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46} &&+|\Psi^{-}\rangle_{13}|\Phi^{-}\rangle_{25}\ \hat{\sigma}_x^4\hat{\sigma}_z^4|\Psi_{\mathrm{out}}\rangle_{46}\\
&+|\Psi^{-}\rangle_{13}|\Psi^{+}\rangle_{25}\ \hat{\sigma}_z^4\hat{\sigma}_x^6\hat{\sigma}_z^6|\Psi_{\mathrm{out}}\rangle_{46} &&+|\Psi^{-}\rangle_{13}|\Psi^{-}\rangle_{25}\ \hat{\sigma}_z^4\hat{\sigma}_x^6|\Psi_{\mathrm{out}}\rangle_{46}\end{aligned} \quad \textbf{[S8]}$$

With the help of polarizing beam splitters, in our experiment we are able to identify the Bell states $|\Phi^{\pm}\rangle_{13}$ and $|\Phi^{\pm}\rangle_{25}$; i.e., we project the combined state of photons 1, 2, 3, and 5 onto one of the four possibilities $|\Phi^{\pm}\rangle_{13}|\Phi^{\pm}\rangle_{25}$. We thus have to consider four different results of the Bell-state measurements (BSMs)—see Table S1. To receive the desired final state of photons 4 and 6, we have to apply corresponding Pauli operations, depending on the outcome of the BSMs.

Teleportation-Based Controlled-Phase (C-Phase) Gate. Similarly to the last section, in the implementation of a teleportation-based C-Phase gate, we also need to apply 16 different Pauli corrections on the output qubits according to 16 possible combinations of outcomes of BSMs. We list the required correction operations in Table S2.

1. Langford NK, et al. (2005) Demonstration of a simple entangling optical gate and its use in Bell-state analysis. *Phys Rev Lett* 95:210504.
2. Kiesel N, et al. (2005) Linear optics controlled-phase gate made simple. *Phys Rev Lett* 95:210505.
3. Okamoto R, Hofmann HF, Takeuchi S, Sasaki K (2005) Demonstration of an optical quantum controlled-not gate without path interference. *Phys Rev Lett* 95:210506.
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5. Gottesman D, Chuang IL (1999) Demonstrating the viability of universal quantum computation using teleportation and single-qubit operations. *Nature* 402:390.

Table S1.

Result of BSMs	Output state
$\|\Phi^{+}\rangle_{13}\|\Phi^{+}\rangle_{25}$	$\|\Psi_{\text{out}}\rangle_{46}$
$\|\Phi^{+}\rangle_{13}\|\Phi^{-}\rangle_{25}$	$\hat{\sigma}_z^6\|\Psi_{\text{out}}\rangle_{46}$
$\|\Phi^{-}\rangle_{13}\|\Phi^{+}\rangle_{25}$	$\hat{\sigma}_z^4\hat{\sigma}_z^6\|\Psi_{\text{out}}\rangle_{46}$
$\|\Phi^{-}\rangle_{13}\|\Phi^{-}\rangle_{25}$	$\hat{\sigma}_z^4\|\Psi_{\text{out}}\rangle_{46}$

Table S2. The required correction operations depending on the BSM results

Measurements	$\|\Phi^{+}\rangle_3$	$\|\Phi^{-}\rangle_3$	$\|\Psi^{+}\rangle_3$	$\|\Psi^{-}\rangle_3$
$\|\Phi^{+}\rangle_5$	I	$Z_{4'}$	$X_{4'}Z_{6'}$	$iY_{4'}Z_{6'}$
$\|\Phi^{-}\rangle_5$	$Z_{6'}$	$Z_{4'}Z_{6'}$	$X_{4'}$	$iY_{4'}$
$\|\Psi^{+}\rangle_5$	$Z_{4'}X_{6'}$	$X_{6'}$	$Y_{4'}Y_{6'}$	$-iX_{4'}Y_{6'}$
$\|\Psi^{-}\rangle_5$	$iZ_{4'}Y_{6'}$	$iY_{6'}$	$-iY_{4'}X_{6'}$	$-X_{4'}X_{6'}$

We define $|\Phi^{\pm}\rangle_i = \frac{1}{\sqrt{2}}(|H\rangle_i|H'\rangle_i \pm |V\rangle_i|V'\rangle_i)$, $|\Psi^{\pm}\rangle_i = \frac{1}{\sqrt{2}}(|H\rangle_i|V'\rangle_i \pm |V\rangle_i|H'\rangle_i)$, where $|H'\rangle$ and $|V'\rangle$ are spatial qubits. The 16 cases in Fig. 6 in the main text correspond to the output results: $|\Phi^{+}\rangle_3|\Phi^{+}\rangle_5$, $|\Psi^{+}\rangle_3|\Phi^{+}\rangle_5$, $|\Phi^{+}\rangle_3|\Psi^{+}\rangle_5$, $|\Psi^{+}\rangle_3|\Psi^{+}\rangle_5$, $|\Phi^{+}\rangle_3|\Phi^{-}\rangle_5$, $|\Psi^{+}\rangle_3|\Phi^{-}\rangle_5$, $|\Phi^{+}\rangle_3|\Psi^{-}\rangle_5$, $|\Psi^{+}\rangle_3|\Psi^{-}\rangle_5$, $|\Phi^{-}\rangle_3|\Phi^{-}\rangle_5$, $|\Psi^{-}\rangle_3|\Phi^{-}\rangle_5$, $|\Phi^{-}\rangle_3|\Psi^{-}\rangle_5$, $|\Psi^{-}\rangle_3|\Psi^{-}\rangle_5$, $|\Phi^{-}\rangle_3|\Phi^{+}\rangle_5$, $|\Psi^{-}\rangle_3|\Phi^{+}\rangle_5$, $|\Phi^{-}\rangle_3|\Psi^{+}\rangle_5$, $|\Psi^{-}\rangle_3|\Psi^{+}\rangle_5$.