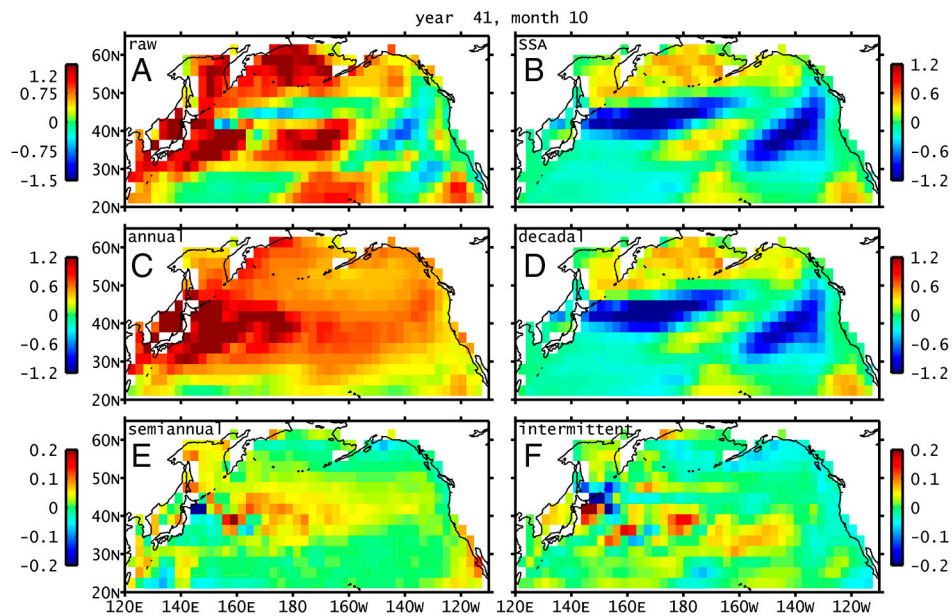


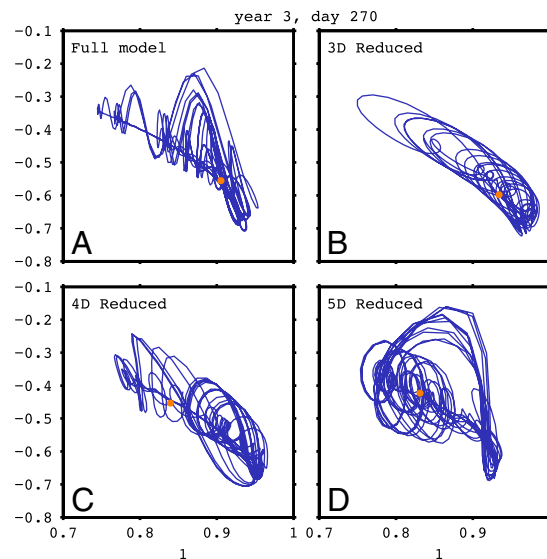
# Supporting Information

Giannakis and Majda 10.1073/pnas.1118984109



**Movie S1.** Spatio-temporal patterns  $\sum_k \bar{x}_k^t$  of the upper 300 m temperature anomaly field (annual mean subtracted at each gridpoint, color-coded in  $^{\circ}\text{C}$ ) evaluated using [9] and [1] for a 100-y portion of the dataset. (A) Raw data. (B) The PDO mode from SSA. (C–F) NLSA patterns using (C) the annual modes,  $k \in \{1, 2\}$  (see Figs. 1 and 2); (D) The leading-low-frequency (PDO) mode,  $k = 3$ ; (E) The semiannual modes,  $k \in \{6, 7\}$ ; (F) The leading two intermittent (Kuroshio) modes,  $k \in \{9, 10\}$ . The starting time of this animation is the same as in Fig. 2. PDO, Pacific Decadal Oscillation; SSA, singular spectrum analysis.

[Movie S1 \(MP4\)](#)



**Movie S2.** 1,000-d trajectories for the zonal modes  $\{x_1, x_4\}$  in (A) the full model (Table S1); (B–D) reduced models constructed by projection onto the NLSA modes  $u_k$  from [9].

[Movie S2 \(MP4\)](#)

**Table S1. Governing equations and parameter values of the six-mode low-order atmospheric model; after Crommelin and Majda (1)**

$$\begin{aligned}
 \dot{x}_1 &= \gamma_1^* x_3 - C(x_1 - x_1^*) \\
 \dot{x}_2 &= -(\alpha_1 x_1 - \beta_1) x_3 - C x_2 - \delta_1 x_4 x_6 \\
 \dot{x}_3 &= (\alpha_1 x_1 - \beta_1) x_2 - \gamma_1 x_1 - C x_3 + \delta_1 x_4 x_5 \\
 \dot{x}_4 &= \gamma_2^* x_6 - C(x_4 - x_4^*) + \epsilon(x_2 x_6 - x_3 x_5) \\
 \dot{x}_5 &= -(\alpha_2 x_1 - \beta_2) x_6 - C x_5 - \delta_2 x_4 x_3 \\
 \dot{x}_6 &= (\alpha_2 x_1 - \beta_2) x_5 - \gamma_2 x_4 - C x_6 + \delta_2 x_4 x_2
 \end{aligned}$$

$$\begin{aligned}
 x_1^* &= 0.95 & x_4^* &= -0.76095 \\
 b &= 0.5 & C &= 0.1 \\
 \beta &= 1.25 & \gamma &= 0.2 \\
 \alpha_m &= \frac{8\sqrt{2}}{\pi} \frac{m^2}{4m^2-1} \frac{b^2+m^2-1}{b^2+m^2} & \beta_m &= \frac{\beta b^2}{b^2+m^2} \\
 \delta_m &= \frac{64\sqrt{2}}{15\pi} \frac{b^2-m^2+1}{b^2+m^2} & \gamma_m^* &= \gamma \frac{4m}{4m^2-1} \frac{\sqrt{2}b}{\pi} \\
 c &= \frac{16\sqrt{2}}{9\pi} & \gamma_m &= \gamma \frac{4m^3}{4m^2-1} \frac{\sqrt{2}b}{\pi(b^2+m^2)}
 \end{aligned}$$

1 Crommelin DT, Majda AJ (2004) Strategies for model reduction: Comparing different optimal bases. *J Atmos Sci* 61:2206–2217.

**Other Supporting Information Files**  
[SI Appendix \(PDF\)](#)