Supporting Information

Li et al. 10.1073/pnas.1309319110

SI Materials and Methods

The Forbidden City and the Heavy Stones. The Forbidden City (i.e., the current Palace Museum), with an area of 753 m × 961 m, was first built during 1406–1420; some palaces were rebuilt after fire accidents in the 15th and 16th centuries (1). The stone carvings used in the construction were white marble stones (~2,500 kg/m^3), which were mined and transported from the Dashiwo quarry in Fangshan over 70 km away [140 Li (2); 1 Li ≈ 500 m]. Both sliding sledges and wheeled vehicles were used to transport the heavy stones, as we learned from the ancient Chinese information recorded in ref. 3, only some of which is found in modern Chinese literature (1): (*i*) The three halls in the outer court were rebuilt in 1557–1562, where the heaviest stone with a size of 9.6 m × 3.2 m × 1.6 m (~123 tons) was transported in 28 d to the site with a sliding sledge hauled by men (case 3 in Table 1), and wells were dug every 1 Li along the road for the water supply for watering and running the sledges and (*ii*) the two palaces in the inner court were rebuilt during 1596–1598, and wheeled vehicles were used in the transportation of heavy stones. The heaviest stone weighed about 95 tons (160,000 Jin; 1 Jin = 593.1 g) and was transported in 22 d to the site with a 16-wheel vehicle hauled by mules. The dates of the two rebuildings were consistent with the information in the official Chinese history of the Ming Dynasty (4).

Large Stone Carving. The description of the Large Stone Carving on the introduction board in the Palace Museum is as follows:

> It is the largest stone carving in the palace, 16.75 meters long, 3.07 meters wide, and 1.7 meters thick, and weighs more than 200 tons, hence the name Large Stone Carving. It was carved out of a huge natural stone in the early Ming Dynasty, when the three main halls were constructed. In 1761 (the 26th year of the Qianlong reign period of Qing Dynasty), the old patterns on the stone were all hewn away, and new patterns were carved. . . . The stone was quarried from Dashiwo in Fangshan in the western suburbs of Beijing. It was transported to the Palace Museum by sprinkling water on the way in winter to make an iced road. Then it was pulled all the way to the Palace Museum along the iced road.

The Large Stone Carving weighs more than 200 tons according to its current size of 16.75 m × 3.07 m × 1.70 m. It had a size of 17.18 m × 3.20 m × 2.08 m (~280 tons) before it was recarved in 1761, so it is commonly believed to have been more than 300 tons before carving when it was first transported to the site in 1407–1420 (5). Obviously, the Large Stone Carving was much larger than the heavy stones transported for the rebuildings in 1557–1562 and 1596–1598. However, in some modern literature (6, 7) information about the transport during the three periods was mixed up.

Wheeled Wagons and Their Load Capacity. In the 15th and 16th centuries, four-wheeled wagons were widely used in transporting heavy loads around 3.5 tons (described in detail in ref. 8). Eight-wheeled wagons were developed around 1538 for transporting stones for the construction of the Ming Tombs (9). During the rebuilding of the two palaces in 1596–1598, as we learned from the ancient Chinese recorded in ref. 3, at least 100 four-wheeled wagons and 20 improved eight-wheeled wagons were hired for transporting the stones, and a 16-wheeled wagon was then specially developed for huge stones that previously were transported by sliding sledges, which indicates that the huge stones, at least the heaviest one of 95 tons, were beyond the load capacity of the eight-wheeled wagons. Therefore, the 123-ton stone transported around 1557 and the 300-ton Large Stone Carving transported around 1407–1420 were well beyond the load capacity of wheeled wagons at the time of the transport.

In addition, transportation with mule-powered wheeled wagons was described (3) as labor-saving, time-saving and low-cost, but it had significant risks of accidents, such as broken wheels, injury to mules, and subsequent damage to the valuable stones being transported. Also, the maintenance of the roads to achieve sufficient flatness and hardness for transportation had been emphasized to avoid accidents (3).

Ice Friction of the Far North Case (Case 4 in Table 1). In the far north case, sledges loaded with mining equipment of a maximum load m = 28 tons were dragged by a track-type tractor with a crew of four men in the ambient temperature of T_a = −55 °C and traveled s = 212.4 km (132 miles) in 10 d in 1934, according to the statement in ref. 10.

Here, we consider heat transfer for the wood-on-ice sliding, which is simplified to an unsteady one-dimensional conduction problem with a constant heat flux input induced by the frictional heating (Fig. S1*A*). The variation of the temperature at the contact surface and the possibility of local melting are investigated. Because of a lack of other detailed information, we make the following assumptions:

(*i*) The dimension of the sledge. We assume that the sledge has a length of l = 4.5 m and a width of w = 3 m, which are typical dimensions of a track-type tractor weighing around 30 tons (e.g., for mining operations) (11). The contact area is $A = lb$, where the width of the contact area is considered as $b = 0.2w$ = 0.6 m as a common sledge.

(*ii*) The sliding velocity of the sledge. Assuming that the sledges were moved continuously in the 10 d with the four men driving alternately, we obtain the time for the sliding t = 240 h, so that the average sliding speed of the sled was $U = s/t \approx$ 25 cm/s.

(*iii*) The temperatures. The original temperature of both the wood sledge and the ice-covered ground are considered uniform T_i = −55 °C, because of the ambient temperature −55 °C. To generate the lubricating water film, the temperature of the ice surface T should first be heated to 0 °C, and then maintained at T = 0 °C until the ice at the surface melts to liquid water, owing to the frictional heating.

(*iv*) The heat flux at the contact surface generated by the frictional force. When the sledge is sliding at a constant velocity, the frictional force $F = \mu mg$ can be considered as the total heat produced per unit displacement (joules per meter), where μ is the coefficient of the friction and $g \approx 10$ m/s^2 is the gravitational acceleration. Then, the total heat energy produced by the frictional force is $Q_F = Fl$. Because the value of μ varies from 0.01–0.43 as reported for experimental results in different sources (12), the maximum energy dissipated that could be produced is $Q_F \approx 5.42 \times 10^5$ J at $\mu \approx 0.43$.

(*v*) The heat transfer model. The ice-covered ground is considered as a semi-infinite solid, and the heat flux $q_F = Q_F/t_l$ at the ice surface generated by the frictional force is considered constant during the contact time $t_l = l/U$ (13).

According to an energy balance, during the contact time heat produced by the frictional force Q_F is equal to the sum of the heat diffusing into the ice-covered ground Q_{ice} and into the wood sledge Q_{wood}, and the heat melting the ice Q_{melt}, shown as

$$Q_F = Q_{ice} + Q_{wood} + Q_{melt}. \quad \textbf{[S1]}$$

In Eq. **S1**, Q_{melt} remains 0 until the temperature at the contact surface reaches 0 °C. The thermal diffusivity of wood $\alpha_{wood} \approx$

1.41×10^{-7} m²/s is much smaller than that of ice $\alpha_{ice} \approx 1.71\times 10^{-6}$ m²/s (around −50 °C), hence we neglect Q_{melt} and Q_{wood} in the estimates below. Therefore, we suppose that all of the heat produced by the frictional force is diffusing into the ice to raise the temperature of ice, so that

$$Q_F \approx Q_{ice} = k_{ice} A(T - T_i)\sqrt{\pi t_l/\alpha_{ice}}/2, \quad \textbf{[S2]}$$

where the heat conductivity of ice $k_{ice} \approx 2.76$ W/(m·°C) at around −50 °C. Therefore, we obtain the temperature at the ice surface can be heated by the frictional heating is $T \approx -29$ °C, which indicates that the wood-on-ice sliding in the far north case is in the dry lubrication regime. Although we could not predict the exact value of μ with the limited information about the case, here we estimate $\mu \approx 0.36$ based on the experimental results of ref. 14, which is measured under the wood-on-ice sliding with a speed of 3 cm/s at −10 °C.

Ice Friction of the Ancient Chinese Case (Case 3 in Table 1). In the ancient Chinese case, sledges loaded with a huge stone of m = 123 tons were dragged by a team of men in the ambient temperature of $T_a = -3.7$ °C, traveling s = 70 km in 28 d in 1557, according to the statement in ref. 3. Because it is estimated that the sliding velocity of the sledge $U = s/t \approx 8$ cm/s, the case was in the low-speed regime, which indicates that a lubricating water film could not be effectively established by the frictional heating according to Bowden's experimental results (14). This conclusion can be verified by the following analysis of heat transfer in sliding.

Because ice can melt to produce lubricant, the coefficient of friction is affected by many factors and varied from 0.01–0.43, as reported for experimental results in different sources (12). However, with the formation of a lubricating water film, the value of μ for wood-on-ice sliding in the high-speed regime (meters per second) is generally about 0.02–0.03 (14). Consequently, here we first suppose that a lubricating water film is established, and then investigate the possibility of the maintenance of the thin film with $\mu \approx 0.03$.

The water-lubricated wood-on-ice sliding is simplified to an unsteady one-dimensional heat conduction problem with a constant surface temperature $T_0 = 0$ °C (Fig. S1*B*), because the thin water film exists at the contact with the ice. The problem is solved with the following assumptions:

(*i*) The dimension of the sledge. We assume that the sledge has a length of $l = 9.6$ m and a width of $w = 3.2$ m, which are the same dimensions as the huge stone being transported (3), and the contact area is $A = lb$, where the width of the contact area is considered as $b = 0.2w = 0.64$ m as a common sledge.

(*ii*) The temperatures. The original temperature of both the wood sledge and the ice-covered ground are considered uniform $T_i = -3.7$ °C, because the ambient temperature is around −3.7 °C.

(*iii*) The typical thickness of the water film is $h = 70$ μm according to the experimental results in ref. 15.

(*iv*) The heat transfer model. Both the ice-covered ground and the wood sledge are considered as a semi-infinite solid (13), and the contact time $t_l = l/U = 120$ s.

According to an energy-balance Eq. **S1**, the heat produced by the frictional force should be able to overcome the heat diffusing into the ice-covered ground and the wooden sledge and melt the ice surface. The heat melting the ice surface per unit area to a water film of $h \approx 70$ μm should be $Q_{melt}/A = L_w \rho_{water} h \approx 2.3 \times 10^4$ J/m², where the latent heat $L_w \approx 3.3 \times 10^5$ J/kg and the density of water $\rho_{water} \approx 1{,}000$ kg/m³.

The heat diffusing into the wooden sledge (or the ice-covered ground) per unit area from $\tau = 0$ to $\tau = t$ is

$$Q/A = \int_0^t q_0/A \, d\tau, \quad \textbf{[S3]}$$

and the heat transfer rate per unit area at the contact surface is

$$q_0/A = \frac{k(T_0 - T_i)}{\sqrt{\pi\alpha\tau}}, \quad \textbf{[S4]}$$

where k is the thermal conductivity and α is the thermal diffusivity. Eq. **S5** can be obtained according to Eqs. **S3** and **S4** as

$$Q/A = 2k(T_0 - T_i)\sqrt{t_l/\pi\alpha}. \quad \textbf{[S5]}$$

Consequently, the heat diffusing into the wooden sledge per unit area Q_{wood} in $t_l = 120$ s can be calculated based on Eq. **S5**, with the characteristics of wood $k_{wood} \approx 0.1$ W/(m·°C), $\alpha_{wood} \approx 1.41\times 10^{-7}$ m²/s, and the result is $Q_{wood}/A \approx 1.2 \times 10^4$ J/m².

Similarly, based on Eq. **S5** the heat diffusing into the ice-covered ground per unit area in $t_l = 120$ s is $Q_{ice}/A \approx 9.2 \times 10^4$ J/m², with the characteristics of ice at 0 °C: $k_{ice} \approx 2.2$ W/(m·°C), $\alpha_{ice} \approx 1.2 \times 10^{-6}$ m²/s.

Because the heat produced by the frictional heating is $Q_F = \mu mgl$, we find that $Q_F/A \approx 5.8 \times 10^4$ J/m² with $\mu \approx 0.03$. Then, the total heat adsorbed per unit area within 120 s is $(Q_{wood} + Q_{ice} + Q_{melt})/A \approx 1.1 \times 10^5$ J/m² > Q_F/A, which indicates that the frictional heating is not able to maintain the thin water film. The result of our estimate agrees with the experimental results in the low-speed regime (14).

Possibility of Maintaining a Water Film Generated by Pouring Water on the Ice-Covered Ground in the Ancient Chinese Case (Case 3 in Table 1). The lubricating water film between the wooden sledge and the ice-covered ground will not freeze completely within the contact time of $t_l = 120$ s according to the following estimates.

Here, we considered the thickness of the layer of the pouring water is $h_{water} = 700$ μm, which is 10 times the thickness of the thin water film generated by frictional heating $h = 70$ μm (15). The heat released per unit area owing to the phase transformation from water to ice is $Q_{pt}/A = L_w \rho_{water} h \approx 2.34 \times 10^5$ J/m², where the latent heat $L_w \approx 3.34 \times 10^5$ J/kg and the density of water $\rho_{water} \approx 1{,}000$ kg/m³. Consequently, the total heat adsorbed per unit area by the wooden sledge and the ice layer within 120 s is $Q_{wood}/A + Q_{ice}/A \approx 1.08 \times 10^5$ J/m² < $Q_{pt}/A \approx 2.34 \times 10^5$ J/m². The result indicates that the thin water film will not freeze within 2 min even without including the heat released by the frictional heating.

The "Getting-Started Problem" of the Sledge Sliding on Ice in the Ancient Chinese Case. In this work, we have mainly focused on the kinetic friction of the ice lubrication in the low-speed regime for heavy-load transportation over a long distance of 70 km. Here are some considerations related to the "getting-started problem" of the sledge.

Regarding the coefficient of the static friction of sliding on ice, it is usually much higher than that of the kinetic friction, although it can became very small with a value around 0.02 at a temperature close to but less than 0 °C when a film of water was present (15).

However, in the ancient Chinese case, the sledge may not get started directly on the ice surface, because it is impractical to park the sledge on the ice-covered ground directly at the end of the day, which will cause the sliding surface to freeze to the ground, after which it would be very hard to restart again. Although we have not found in the literature how the ancient people solved the problem, it is a reasonable solution to park the sledge with the huge stone on planks or small rollers to prevent the sliding surface from freezing to the ground, because this is what modern drivers are instructed to do in cold winter (16, 17). More effort and more workers may be needed to park and restart the sledge, but the force needed should be significantly smaller than that needed to separate the frozen sledge from the ice-covered ground.

Translations of a Related Portion from the Ancient Chinese Book *Liang Gong Ding Jian Ji* (3), First Published *ca*. A.D. 1618. The 24th year of Wanli, [The 24th year of Wanli is A.D. 1596. The two palaces of Qianqing and Kunning in the inner court of the Forbidden City burned down in an accidental fire.] the two palaces of Qianqing and Kunning was planned to be rebuilt. Mr. Shengrui He was assigned to be in charge of the reconstruction. Mr. He realized that the reconstruction was very important, and the amount of work and the cost was enormous. At the beginning of the planning, he felt so much work was involved in the project that it was hard to make a comprehensive plan for it. So he reviewed all of the related documents about the constructions, which were finished in the past, as his references. However, many documents were lost, which made him anxious. Then he found the memorials since the reconstruction of the three halls in the outer court in the 36th year of Jiajing, [The 36th year of Jiajing is A.D. 1557. The three halls of the outer court of the Forbidden City, which are currently named the Hall of Supreme Harmony (Taihe Dian), Hall of Central Harmony (Zhonghe Dian), and Hall of Preserving Harmony (Baohe Dian), burned down in an accidental fire, and reconstruction was completed in the 41st year of Jiangjing (A.D. 1562).] which were kept in the Ministry of Works, to read and make detailed notes.

The reconstruction of the two palaces of Qianqing and Kunning were started on July 10th, the 24th year of Wanli, and completed on July 15th, the 26th year.

The huge stones used in the reconstruction of the three halls in the outer court were mined from the Dashiwo Quarry.

The huge stones, which were used along the central way of the three halls, had a length of 3 Zhang, width of 1 Zhang and thickness of 5 Chi. [The unit of length 1 Zhang = 10 Chi ≈ 3.2 m.] Sledges were made and hauled by hired peasants to transport these stones, and 20,000 men from eight districts were hired and involved in the entire transportation. Two officers were in charge. Wells were dug every 1 Li [In the Ming Dynasty 1 Li = 180 Zhang ≈ 576 m, usually it is estimated as 1 Li ≈ 500 m.] to supply the water for watering and running the sledge, and also for drinking water. It took 28 d for the stones to be transported to the Forbidden City, and the total cost was more than 110,000 taels of silver. [A tael of sliver was a unit of currency in ancient China.] When planning the transportation of the huge stones for the reconstruction of the two palaces, Mr. Jingchen Liu had suggested following the previous method of transportation and hiring masses of men from five districts nearby. However, Mr. Shengrui He adopted Mr. Zhiyi Guo's suggestion, which was to develop 16-wheeled wagons for transportation. There were 1,800 mules involved in towing all of the wagons, and it took 22 d to transport the stones to the Forbidden City, and the total cost was less than 7,000 taels of silver.

According to the information provided by Mr. Zhiyi Guo, the government prepared 100 four-wheeled wagons. ... As for improved eight-wheeled wagons, there were only three kept at the gate of Xihuamen at that time, and 17 more were manufactured at a cost of 50 taels of silver per wagon. If these wagons were still not enough, more wagons could be prepared when needed.

After the winter solstice, it was the coldest time with cold weather and hard ground, so that it was the best time for the transportation of the huge stones.

Regarding payment for the transportation of the huge stones, the stones with a volume less than 22 Zhang [The unit of volume 1 Zhang ≈ 0.33 m^3 (2).] were counted as per day per mule, and the price was already proved. However, there were many huge stones with a volume above 22 Zhang and up to 80–90 Zhang. If the transportation work was assigned as a duty to ordinary men or recruited people, they could not afford the loss when there was an accident in transit. Therefore, some officers were assigned to take care of transporting the stones with a volume from 40 Zhang up to 90 Zhang, and the hired men were paid as per day per mule. If there were accidents such as broken wagon wheels or wounded mules, the officers could deal with the corresponding compensation.

Maintenance of the road was necessary. There were several huge stones, some of which weighed 150,000–160,000 Jin as a single piece, and some weighed more than 100,000 Jin. The cost of mining such stones was more than 1,000 taels of silver each. If the road is not sufficiently flat and hard, damage to the wheeled wagons and subsequent damage of the stones would likely happen. Therefore, the road should be properly maintained to sufficient width, hardness and flatness to ensure the accomplishment of the transportation on time.

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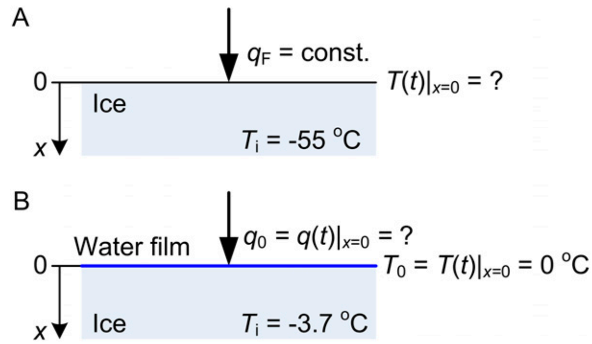

Fig. S1. The heat transfer models. (*A*) Model for the far north case: unsteady one-dimensional conduction problem with a constant heat flux input. (*B*) Model for the ancient Chinese case: unsteady one-dimensional conduction problem with a constant surface temperature.