Quantum mechanics writ large

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Some two centuries before the quantum revolution, Newton (1) suggested that corpuscles of light generate waves in an aethereal medium like skipping stones generate waves in water, with their motion then being affected by these aether waves. Times have changed. Light corpuscles are now known as photons, and the majority of physicists have dispensed with the notion of aether. Nevertheless, certain features of Newton's metaphor live on in one particular version of quantum mechanics. According to pilot wave theory, first proposed by de Broglie (2) and later developed by Bohm (3) with Einstein's encouragement, microscopic elements such as photons and electrons consist of both particle and wave, the former being guided by the latter. Although this physical picture has not been widely accepted, it has had some notable proponents, including Bell (4). Its principal appeal is that it restores realism and determinism to quantum mechanics, its weakness that the physical nature of the guiding wave field remains unclear. At the time that pilot wave theory was developed and then overtaken by the Copenhagen interpretation as the standard view of quantum mechanics, there was no macroscopic pilot wave analog to draw upon. Now there is.

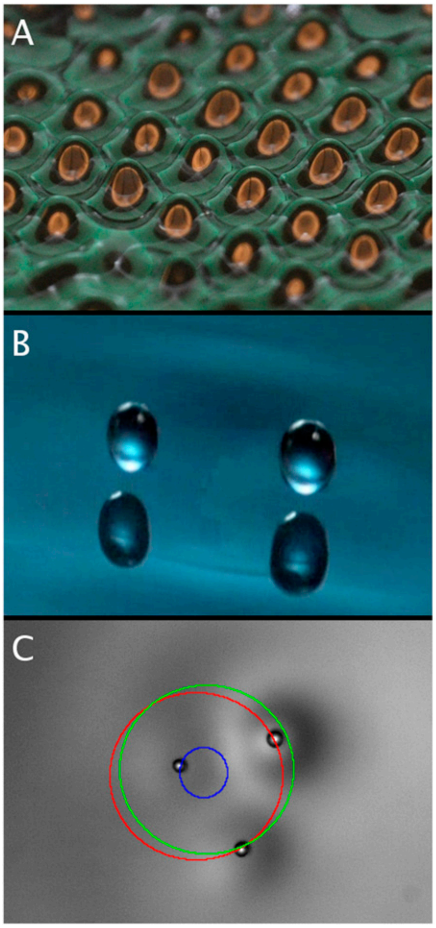

Fig. 1. (*A*) Millimetric Faraday waves are generated on the surface of silicone oil by driving the fluid layer vertically in a periodic fashion above the Faraday threshold. (*B*) A pair of droplets (diameter = 1 mm) on the surface of silicone oil, above the bouncing threshold but below the Faraday threshold. The droplets communicate through their wave fields, locking into phase at a fixed distance apart. (*C*) Three walking droplets lock into a collective orbit. All orbits are counterclockwise, the largest being 8 mm in diameter.

"Path-memory induced quantization of classical orbits" [Fort et al. (5)] is the latest in a remarkable series of papers by Couder and coworkers (6–12), who have discovered a macroscopic pilot wave system that exhibits several features previously thought to be peculiar to the microscopic realm. When a fluid bath is driven up and down in a periodic fashion, there is a critical acceleration that depends on the fluid viscosity, depth, and surface tension, below which the interface remains horizontal and above which the surface goes unstable to a regular pattern of millimetric Faraday waves (Fig. 1*A*) whose period is twice that of the forcing (13, 14). When a droplet of characteristic diameter 1 mm is placed on the vibrating surface of a fluid bath, it may lift off provided that the vertical acceleration of the free surface exceeds that due to gravity. When it lands, it can avoid coalescing provided that the impact time is less than the time required for the air layer between the drop and bath to drain to some critical distance at which merger is initiated by van der Waals forces. The experiments of Couder involve placing a droplet on a bath of silicone oil (with viscosity 20–50 times that of water) in a driving regime above the bouncing threshold but below the Faraday threshold. There, an appropriately sized drop may bounce indefinitely on the free surface, generating a localized field of surface waves that decays with distance from the drop (6). If multiple bouncers are placed on the free surface, they communicate through their wave fields (Fig. 1*B*). An assemblage of equal-sized bouncers may lock into lattices corresponding to Archimedean tilings (7). Neighboring bouncers of unequal size may lock together and ratchet across the free surface in pairs or form larger aggregates of rotating, drifting rafts (8).

At the bouncing threshold, a single droplet will bounce with the forcing period. Increasing the driving amplitude eventually prompts a period doubling transition, after which the period of the bouncer becomes twice that of the driving and so commensurate with that of Faraday waves (9, 10). The waves generated by the bouncer may then destabilize the vertical bouncing state. If slightly perturbed in a given direction, the drop lands on a sloping interface and so is nudged in that same direction. Remarkably, the droplet can thus walk in a steady fashion across the surface, being piloted along at each step by its wave field (9). A simple time-averaged model for the walker dynamics and criterion for walking was developed by Protière et al. (10). Walking is only possible when the guiding wave field is large (that is, as the Faraday threshold is approached); consequently, drops walk more readily in deeper fluid. As in the case of bouncers, multiple walkers interact through their wave fields: an approaching pair of walking droplets may either scatter, lock into orbit, or coalesce (Fig. 1*C*).

To explore the wave-particle nature of the walking droplets, Couder and Fort (11) examined their behavior as they passed through obstructions; specifically, they undertook macroscopic versions of single-particle single- and double-slit experiments. In their experiments, the walkers were directed toward a slit, specifically, a gap in a subsurface barrier that reduced the fluid depth below that required for walking. In the single-slit experiment, they found that the walker's path was deflected owing to the interaction of its wave field with the barrier. Repetition of the experiment revealed the emergence of a diffraction pattern in the distribution of droplet trajectories; thus, their experiments are a macroscopic analog of the classic single-photon diffraction experiments of Taylor (15). In the double-slit experiment, repetition of the experiments revealed the emergence of interference patterns: while the drop passed through one slit or the other, its accompanying wave passed through both, and

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See companion article on page 17515.

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the wave interference led to the interference pattern apparent in the distribution of droplet trajectories.

The double-slit experiment (16) holds a central place in the development of quantum theory. In the words of Feynman et al. (17), it is "impossible, absolutely impossible to explain in any classical way, and which has in it the heart of quantum mechanics." Indeed, unless one ascribes to pilot wave theory, an unsettling feature of the experiment is that interference patterns persist even when the electrons pass through the slit one at a time, an effect shown to persist with particles as large as fullerenes (18). Couder and Fort (11) showed that pilot waves readily produce such an effect on a macroscopic scale, with droplets a million times larger than fullerenes. Another peculiar feature of the single-electron double-slit experiment is that if one observes which slit the electron passes through, the interference pattern vanishes (16). Of course, owing to the enormous difference in scale between the droplets and the photons that allow us to see them, there is no such measurement problem in the experiments of Couder and Fort (11): one can readily observe the fate of both droplet and wave. Nevertheless, it is not difficult to imagine a measurement technique so heavy-handed that it would disturb the free surface sufficiently to destroy the interference pattern (for example, if the drops could only be seen by their effect on a stream of droplets impinging on the two slits). Finally, it is noteworthy that the droplet diffraction system presumably exhibits statistical behavior not because it is intrinsically probabilistic, but because the interaction between the droplet, its wave field, and the slit is sufficiently complex to render the system sensitive to initial conditions.

Eddi et al. (12) examined the interaction of a walker with a barrier and so developed a macroscopic version of quantum tunneling (19). Tunneling arises in quantum mechanics when a microscopic particle beats the odds by crossing a barrier, an effect that has recently led to a new generation of microscopes; for example, the scanning tunneling microscope has yielded unprecedented insights into the dynamics of electrons in confined geometries (20). In the experiments of Eddi et al. (12), walkers are confined by four walls corresponding to vertical barriers, above which the droplets can bounce but not walk. The incidence of the walker's wave field on the barrier leads to partial reflection and an evanescent tail that decays across the barrier. The reflected wave typically causes an approaching walker to be reflected from the barrier; however, the particle-wave–barrier interaction does occasionally permit the droplet to tunnel across. The tunneling probability is shown to decrease with the width of the wall and increase as the Faraday threshold is approached (12). Once again, the analog quantum behavior is caused by the interaction between the droplet and its guiding wave.

In their latest contribution, Fort et al. (5) examine the dynamics of droplets walking in a rotating frame. One expects the walkers to follow a circular orbit on which the radially outward centripetal force and the inward Coriolis force balance, the radius of which is given by $R = V/2\Omega$, where V is the walking speed and Ω the rotation rate. Although such is the case far below the Faraday threshold, the orbits become quantized as this threshold is approached and the wave field becomes more pronounced. Once again, the anomalous quantum behavior is associated with the interaction of the droplet with its wave field, which the authors refer to as its path memory. Their numerical model captures the interaction between the droplet and its wave field and allows them to rationalize the orbital quantization arising when the walker's memory is deep. In an elegant theoretical development, they show that the effective wave force in this deep-memory limit is equivalent to that generated by a single image droplet on the opposite side of the orbit. The authors draw a provocative analogy with Landau levels, the quantized orbits arising in quantum mechanics when a charge translates in a magnetic field, pointing out the nearly exact equivalence of the two systems when the Faraday wavelength is identified with the de Broglie wavelength.

One is currently taught that the macroscopic and microscopic worlds are intrinsically different, the former being deterministic and the latter probabilistic. By virtue of its wave particle nature, the walking drop exhibits several features previously thought to be peculiar to the microscopic realm, including single-particle diffraction, interference, tunneling, and now, quantized orbits. These studies raise a number of fascinating questions. Are the macroscopic and microscopic worlds really so different? Might the former yet yield insight into the latter? Is there really a connection between this bouncing droplet system and the microscopic world of subatomic particles? Or is it all just an odd coincidence? By virtue of its accompanying pilot wave, the walker's dynamics are temporally nonlocal, depending on its bouncing history, its memory. Indeed, this memory is responsible for all of their anomalous quantum behavior. Might such nonlocality give rise to something equivalent to entanglement, one of the central mysteries of modern quantum theory? Might it be possible to impart to the walkers an attribute equivalent to quantum spin? When this exciting line of research has run its course, what dynamical features will remain exclusive to the microscopic world? Time will tell. One thing, though, is certain. In physics, as in life, things would be much simpler, but far less interesting, were it not for the depth of memory.

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